Learning Automated Intrusion Response Ericsson Research

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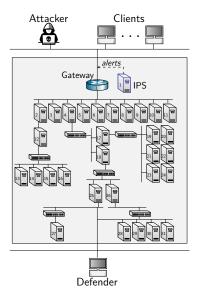
December 8, 2023



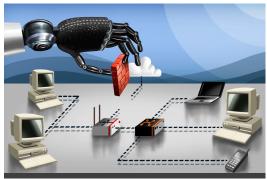
Use Case: Intrusion Response

A defender owns an infrastructure

- Consists of connected components
- Components run network services
- Defender defends the infrastructure by monitoring and active defense
- Has partial observability
- An attacker seeks to intrude on the infrastructure
 - Has a partial view of the infrastructure
 - Wants to compromise specific components
 - Attacks by reconnaissance, exploitation and pivoting



Automated Intrusion Response



Levels of security automation









No automation. Manual detection. Manual prevention. Lack of tools.

Operator assistance. Audit logs Manual detection. Manual prevention. Partial automation.

Manual configuration. Intrusion detection systems. Intrusion prevention systems.

High automation.

System automatically updates itself.

1980s

1990s

2000s-Now

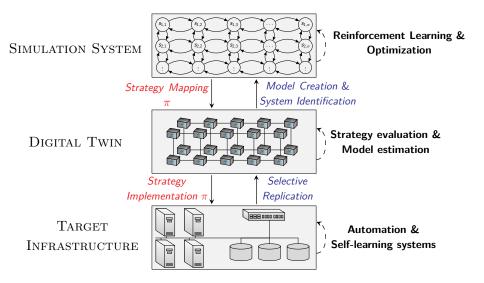
Research

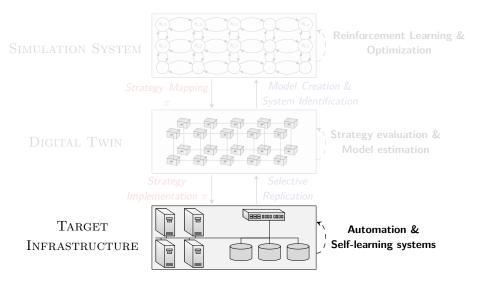
Automated Intrusion Response

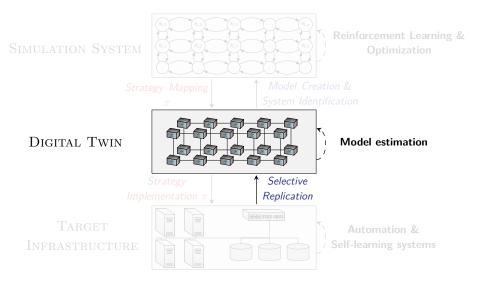


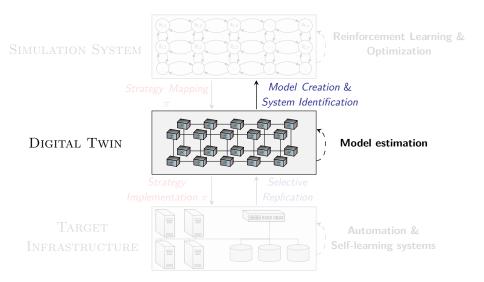
Can we find effective security strategies through decision-theoretic methods?

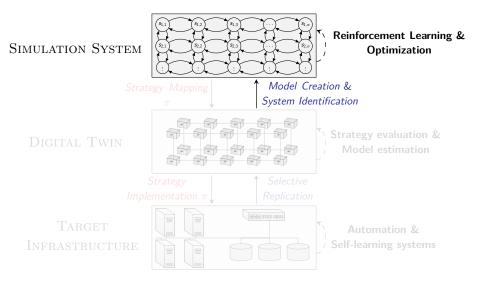


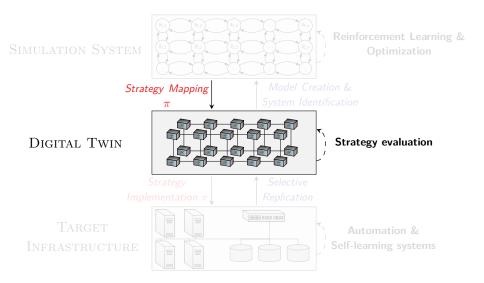


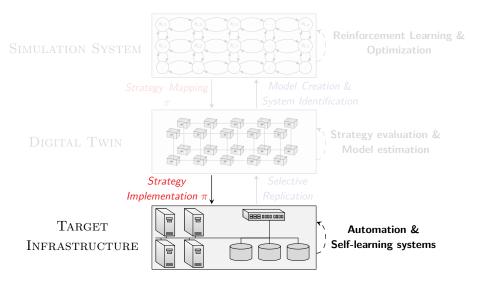


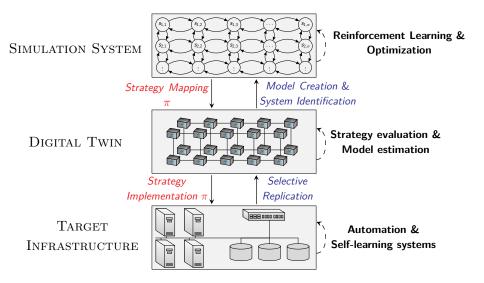




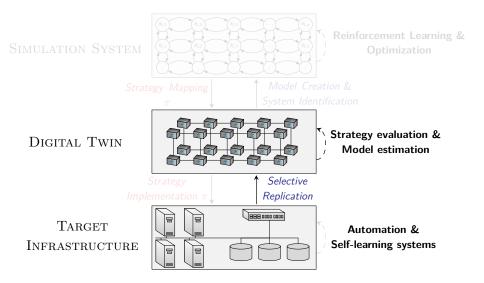




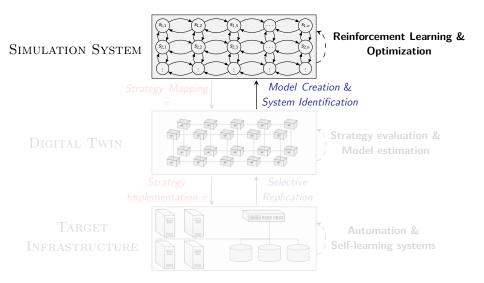




Creating a Digital Twin of the Target Infrastructure



Learning of Defender Strategies



Example Infrastructure Configuration

▶ 64 nodes

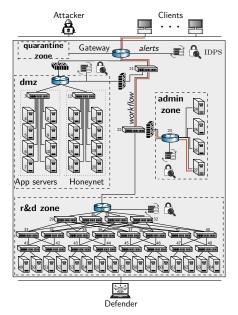
- 24 OVS switches
- 3 gateways
- 6 honeypots
- 8 application servers
- 4 administration servers
- 15 compute servers

11 vulnerabilities

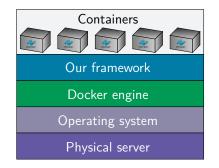
- CVE-2010-0426
- CVE-2015-3306
- etc.

Management

- 1 SDN controller
- 1 Kafka server
- 1 elastic server



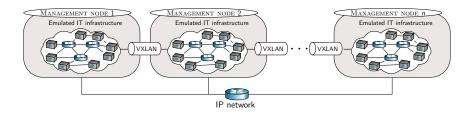
Emulating Physical Components



We emulate physical components with Docker containers

- Focus on linux-based systems
- Our framework provides the orchestration layer

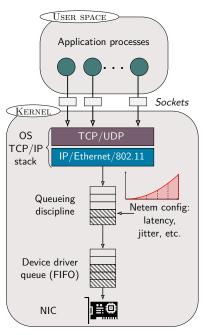
Emulating Network Connectivity

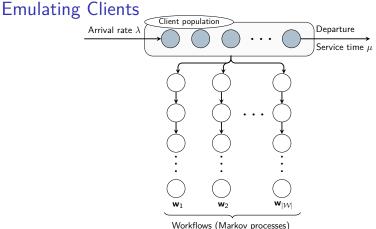


- We emulate network connectivity on the same host using network namespaces
- Connectivity across physical hosts is achieved using VXLAN tunnels with Docker swarm

Emulating Network Conditions

- Traffic shaping using NetEm
- Allows to configure:
 - Delay
 - Capacity
 - Packet Loss
 - Jitter
 - Queueing delays
 - etc.





- Homogeneous client population
- Clients arrive according to $Po(\lambda)$
- Client service times $Exp(\mu)$
- Service dependencies $(S_t)_{t=1,2,...} \sim MC$

Emulating The Attacker and The Defender

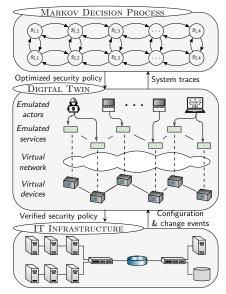
 API for automated defender and attacker actions

Attacker actions:

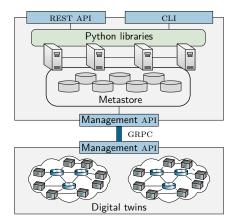
- Exploits
- Reconnaissance
- Pivoting
- etc.

Defender actions:

- Shut downs
- Redirect
- Isolate
- Recover
- Migrate
- etc.



Software framework

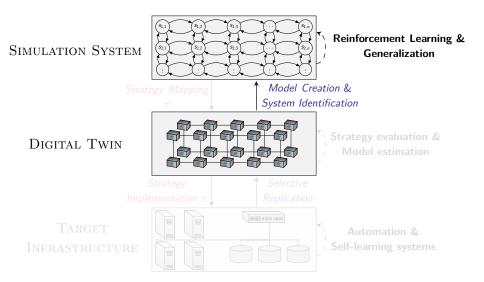


More details about the software framework

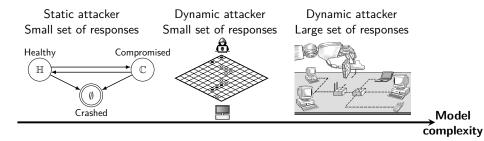
Source code: https://github.com/Limmen/csle

- Documentation: http://limmen.dev/csle/
- Demo: https://www.youtube.com/watch?v=iE2KPmtIs2A

System Identification



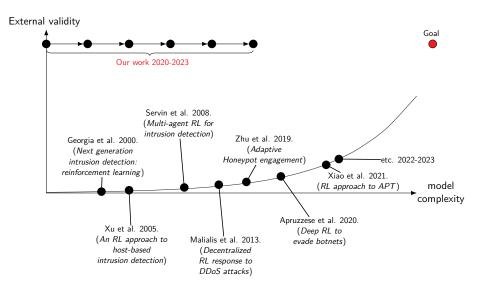
System Model



Intrusion response can be modeled in many ways

- As a parametric optimization problem
- As an optimal stopping problem
- As a dynamic program
- As a game
- etc.

Related Work on Learning Automated Intrusion Response

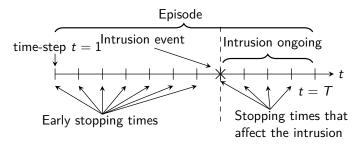


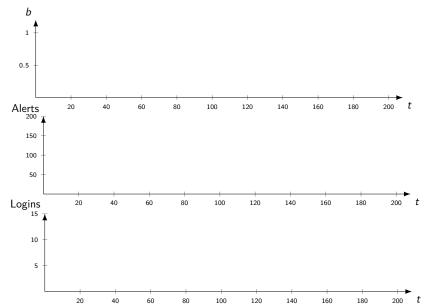
Intrusion Response through Optimal Stopping

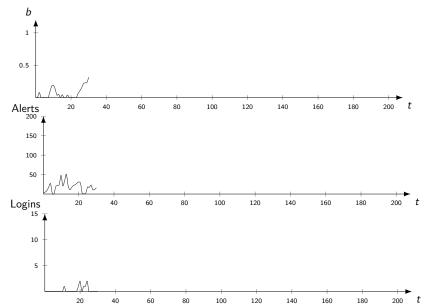
Suppose

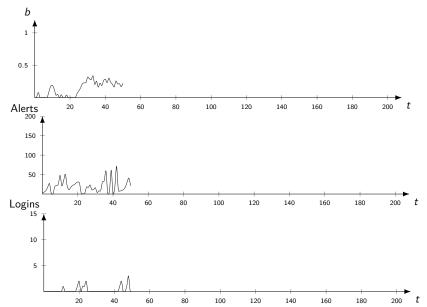
- The attacker follows a fixed strategy (no adaptation)
- We only have one response action, e.g., block the gateway

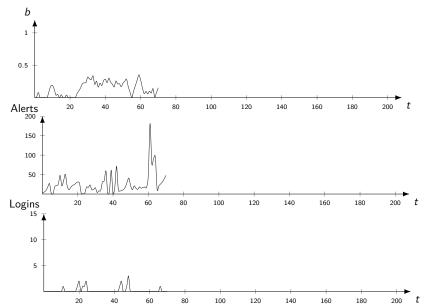
Formulate intrusion response as optimal stopping

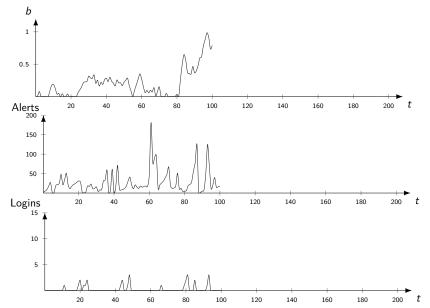


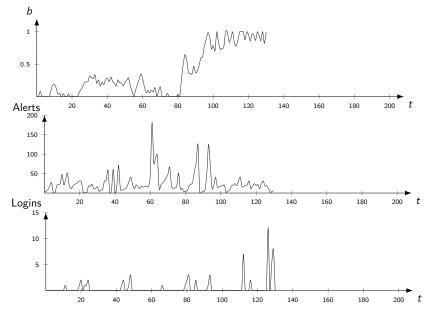


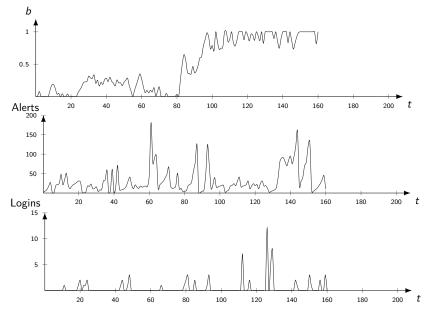


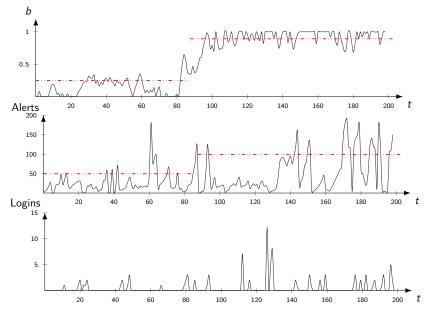


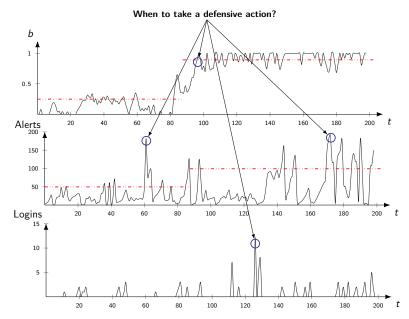










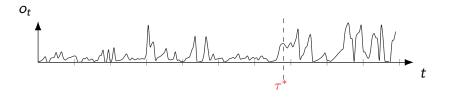


The Defender's Optimal Stopping Problem (1/3)

- lnfrastructure is a discrete-time dynamical system $(s_t)_{t=1}^T$
- Defender observes a noisy observation process $(o_t)_{t=1}^T$
- Two options at each time t: (C)ontinue and (S)stop
- Find the optimal stopping time τ^* :

$$\tau^{\star} \in \arg\max_{\tau} \mathbb{E}_{\tau} \bigg[\sum_{t=1}^{\tau-1} \gamma^{t-1} \mathcal{R}^{\mathfrak{C}}_{s_{t}s_{t+1}} + \gamma^{\tau-1} \mathcal{R}^{\mathfrak{S}}_{s_{\tau}s_{\tau}} \bigg]$$

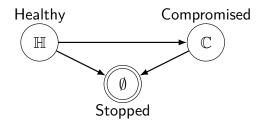
where $\mathcal{R}_{ss'}^{\mathfrak{S}}$ & $\mathcal{R}_{ss'}^{\mathfrak{C}}$ are the stop/continue rewards and τ is $\tau = \inf\{t : t > 0, a_t = \mathfrak{S}\}$



The Defender's Optimal Stopping Problem (2/3)

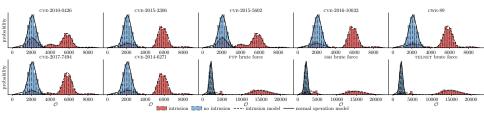
• **Objective:** stop the attack as soon as possible

• Let the state space be $\mathcal{S} = \{\mathbb{H}, \mathbb{C}, \emptyset\}$



The Defender's Optimal Stopping Problem (3/3)

• Let the observation process $(o_t)_{t=1}^T$ represent IDS alerts



- Estimate the observation distribution based on M samples from the twin
- E.g., compute empirical distribution 2 as estimate of Z
 2 →^{a.s} Z as M → ∞ (Glivenko-Cantelli theorem)

Optimal Stopping Strategy

► The defender can compute the **belief**

$$b_t \triangleq \mathbb{P}[S_{i,t} = \mathbb{C} \mid b_1, o_1, o_2, \dots o_t]$$

• Stopping strategy: $\pi(b): [0,1] \to \{\mathfrak{S},\mathfrak{C}\}$

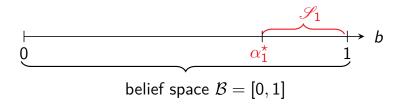
Optimal Threshold Strategy

Theorem

There exists an optimal defender strategy of the form:

$$\pi^{\star}(b) = \mathfrak{S} \iff b \ge \alpha^{\star} \qquad \qquad \alpha^{\star} \in [0, 1]$$

i.e., the stopping set is $\mathscr{S} = [\alpha^*, 1]$



Optimal Multiple Stopping

- Suppose the defender can take $L \ge 1$ response actions
- Find the optimal stopping times $\tau_L^*, \tau_{L-1}^*, \ldots, \tau_1^*$:

$$(\tau_l^{\star})_{l=1,\ldots,L} \in \operatorname*{arg\,max}_{\tau_1,\ldots,\tau_L} \mathbb{E}_{\tau_1,\ldots,\tau_L} \left[\sum_{t=1}^{\tau_L-1} \gamma^{t-1} \mathcal{R}_{s_t s_{t+1}}^{\mathfrak{C}} + \gamma^{\tau_L-1} \mathcal{R}_{s_{\tau_L} s_{\tau_L}}^{\mathfrak{S}} + \right]$$

$$\sum_{t=\tau_{L}+1}^{\tau_{L-1}-1} \gamma^{t-1} \mathcal{R}_{s_{t}s_{t+1}}^{\mathfrak{C}} + \gamma^{\tau_{L-1}-1} \mathcal{R}_{s_{\tau_{L-1}}s_{\tau_{L-2}}}^{\mathfrak{S}} + \dots + \sum_{t=\tau_{2}+1}^{\tau_{1}-1} \gamma^{t-1} \mathcal{R}_{s_{t}s_{t+1}}^{\mathfrak{C}} + \gamma^{\tau_{1}-1} \mathcal{R}_{s_{\tau_{1}}s_{\tau_{1}}}^{\mathfrak{S}} \bigg]$$

where τ_l denotes the stopping time with l stops remaining.

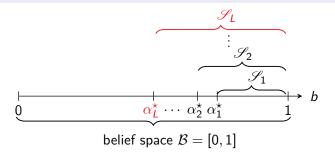
Optimal Multi-Threshold Strategy

Theorem

- Stopping sets are nested $\mathscr{S}_{l-1} \subseteq \mathscr{S}_l$ for $l = 2, \ldots L$.
- If (o_t)_{t≥1} is totally positive of order 2 (TP2), there exists an optimal defender strategy of the form:

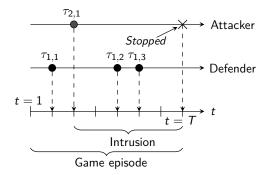
$$\pi_l^{\star}(b) = \mathfrak{S} \iff b \ge \alpha_l^{\star}, \qquad l = 1, \dots, L$$

where $\alpha_l^{\star} \in [0, 1]$ is decreasing in *l*.



Optimal Stopping Game

Suppose the attacker is dynamic and decides when to start and abort its intrusion.



Find the optimal stopping times

 $\max_{\tau_{\mathrm{D},1},...,\tau_{\mathrm{D},L}} \min_{\tau_{\mathrm{A},1},\tau_{\mathrm{A},2}} \mathbb{E}[J]$

where J is the defender's objective.

Best-Response Multi-Threshold Strategies (1/2)

Theorem

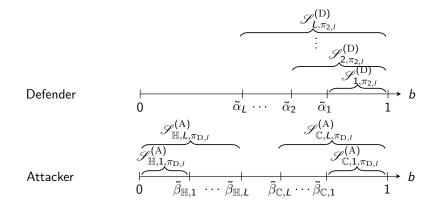
• The defender's best response is of the form:

$$ilde{\pi}_{\mathrm{D},l}(b) = \mathfrak{S} \iff b \geq ilde{lpha}_l, \qquad l = 1, \dots, L$$

► The attacker's best response is of the form:

$$egin{aligned} & ilde{\pi}_{\mathrm{A},l}(b) = \mathfrak{C} \iff & ilde{\pi}_{\mathrm{D},l}(\mathfrak{S} \mid b) \geq & ilde{eta}_{\mathbb{H},l}, \quad l = 1, \dots, L, s = \mathbb{H} \ & ilde{\pi}_{\mathrm{A},l}(b) = \mathfrak{S} \iff & ilde{\pi}_{\mathrm{D},l}(\mathfrak{S} \mid b) \geq & ilde{eta}_{\mathbb{C},l}, \quad l = 1, \dots, L, s = \mathbb{C} \end{aligned}$$

Best-Response Multi-Threshold Strategies (2/2)

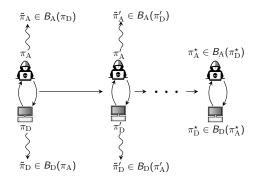


Efficient Computation of Best Responses

Algorithm 1: Threshold Optimization

- 1 **Input:** Objective function *J*, number of thresholds *L*, parametric optimizer PO
- 2 **Output:** A approximate best response strategy $\hat{\pi}_{ heta}$
- - Examples of parameteric optimization algorithmns: CEM, BO, CMA-ES, DE, SPSA, etc.

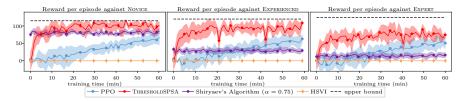
Threshold-Fictitious Play to Approximate an Equilibrium

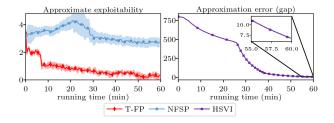


Fictitious play: iterative averaging of best responses.

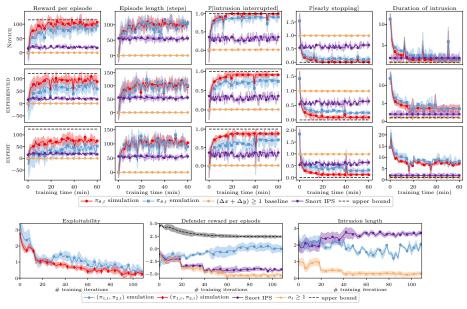
- **Learn best response** strategies iteratively
- Average best responses to approximate the equilibrium

Comparison against State-of-the-art Algorithms

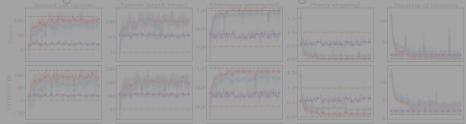




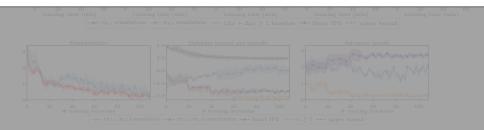
Learning Curves in Simulation and Digital Twin



Learning Curves in Simulation and Digital Twin

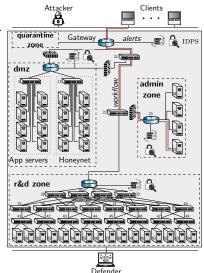


Stopping is about **timing**; now we consider timing + action selection



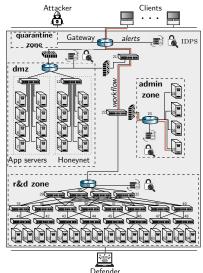
General Intrusion Response Game

- Suppose the defender and the attacker can take L actions per node
- G = ⟨{gw} ∪ V, E⟩: directed tree representing the virtual infrastructure
- ► V: set of virtual nodes
- \blacktriangleright \mathcal{E} : set of node dependencies
- ► Z: set of zones



General Intrusion Response Game

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State Space

• Each $i \in \mathcal{V}$ has a state

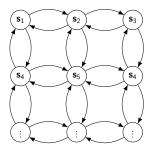
$$\mathbf{v}_{i,t} = (\underbrace{\mathbf{v}_{t,i}^{(\mathrm{Z})}}_{\mathrm{D}}, \underbrace{\mathbf{v}_{t,i}^{(\mathrm{I})}, \mathbf{v}_{t,i}^{(\mathrm{R})}}_{\mathrm{A}})$$

• System state
$$\mathbf{s}_t = (\mathbf{v}_{t,i})_{i \in \mathcal{V}} \sim \mathbf{S}_t$$

 Markovian time-homogeneous dynamics:

$$\mathbf{s}_{t+1} \sim f(\cdot \mid \mathbf{S}_t, \mathbf{A}_t)$$

 $\mathbf{A}_t = (\mathbf{A}_t^{(\mathrm{A})}, \mathbf{A}_t^{(\mathrm{D})})$ are the actions.



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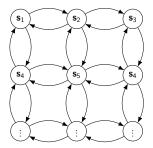
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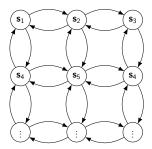
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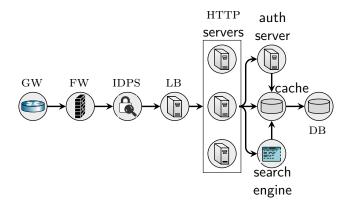
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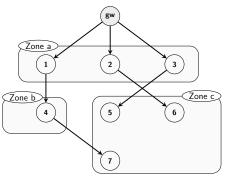
Services are connected into workflows $\mathcal{W} = \{\mathbf{w}_1, \dots, \mathbf{w}_{|\mathcal{W}|}\}.$

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- ► Each w ∈ W is realized as a subtree G_w = ⟨{gw} ∪ V_w, E_w⟩ of G

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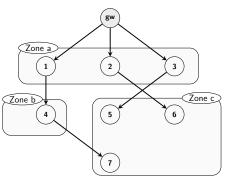
A workflow tree

 $\mathcal{V} = igcup_{\mathsf{w}_i \in \mathcal{W}} \mathcal{V}_{\mathsf{w}_i} ext{ such that } i
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partitioning



A workflow tree

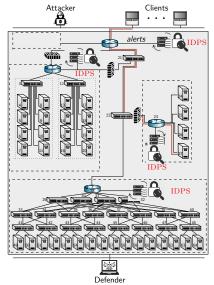
$$\mathcal{V} = \bigcup_{\mathbf{w}_i \in \mathcal{W}} \mathcal{V}_{\mathbf{w}_i} \text{ such that } i \neq j \implies \mathcal{V}_{\mathbf{w}_i} \cap \mathcal{V}_{\mathbf{w}_j} = \emptyset$$

Observations

IDPSs inspect network traffic and generate alert vectors:

$$\mathbf{o}_t \triangleq \left(\mathbf{o}_{t,1}, \dots, \mathbf{o}_{t,|\mathcal{V}|}\right) \in \mathbb{N}_0^{|\mathcal{V}|}$$

- $\mathbf{o}_{t,i}$ is the number of alerts related to node $i \in \mathcal{V}$ at time-step t.
- ▶ o_t = (o_{t,1},..., o_{t,|V|}) is a realization of the random vector O_t with joint distribution Z



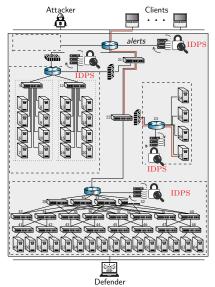
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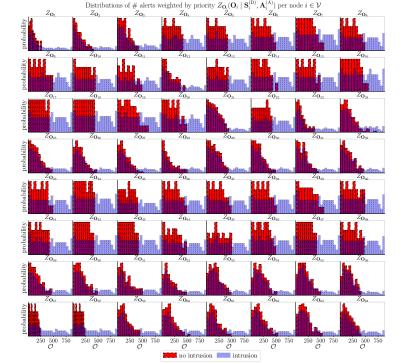
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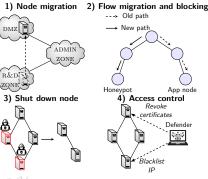


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Defender

- Defender action: $\mathbf{a}_{t}^{(D)} \in \{0, 1, 2, 3, 4\}^{|\mathcal{V}|}$
- ▶ 0 means do nothing. 1 4 correspond to defensive actions (see fig)
- A defender strategy is a function
 - $\mathbf{h}_{t}^{(D)} = (\mathbf{s}_{1}^{(D)}, \mathbf{a}_{1}^{(D)}, \mathbf{o}_{1}, \dots, \mathbf{a}_{t-1}^{(D)}, \mathbf{s}_{t-1}^{(D)}, \mathbf{o}_{t}) \in \mathcal{H}_{D}$
- Objective: (i) maintain workflows; and

$$J \triangleq \sum_{t=1}^{T} \gamma^{t-1} \left(\underbrace{\eta \sum_{i=1}^{|\mathcal{W}|} u_{\mathrm{W}}(\mathbf{w}_{i}, \mathbf{s}_{t})}_{\text{workflows utility}} - \underbrace{(1-\eta) \sum_{j=1}^{|\mathcal{V}|} c_{\mathrm{I}}(\mathbf{s}_{t,j}, \mathbf{a}_{t,j})}_{\text{intrusion and defense costs}} \right)$$



DMZ

Defender

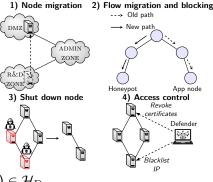
- Defender action: $\mathbf{a}_t^{(D)} \in \{0, 1, 2, 3, 4\}^{|\mathcal{V}|}$
- 0 means do nothing. 1 4 correspond to defensive actions (see fig)
- ► A defender strategy is a function $\pi_{\rm D} \in \Pi_{\rm D} : \mathcal{H}_{\rm D} \to \Delta(\mathcal{A}_{\rm D})$, where

$$\mathbf{h}_t^{(\mathrm{D})} = (\mathbf{s}_1^{(\mathrm{D})}, \mathbf{a}_1^{(\mathrm{D})}, \mathbf{o}_1, \dots, \mathbf{a}_{t-1}^{(\mathrm{D})}, \mathbf{s}_t^{(\mathrm{D})}, \mathbf{o}_t) \in \mathcal{H}_{\mathrm{D}}$$

Objective: (i) maintain workflows; and (ii) stop a possible intrusion:

$$J \triangleq \sum_{t=1}^{T} \gamma^{t-1} \left(\underbrace{\eta \sum_{i=1}^{|\mathcal{W}|} u_{\mathrm{W}}(\mathbf{w}_{i}, \mathbf{s}_{t})}_{\text{workflows utility}} - \underbrace{(1-\eta) \sum_{j=1}^{|\mathcal{V}|} c_{\mathrm{I}}(\mathbf{s}_{t,j}, \mathbf{a}_{t,j})}_{\text{intrusion and defense costs}} \right)$$





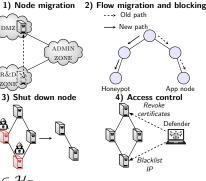
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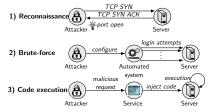
R&D

Attacker

- Attacker action: $\mathbf{a}_t^{(A)} \in \{0, 1, 2, 3\}^{|\mathcal{V}|}$
- 0 means do nothing. 1 3 correspond to attacks (see fig)
- An attacker strategy is a function π_A ∈ Π_A : ℋ_A → Δ(𝒫_A), where ℋ_A is the space of all possible attacker histories

$$\mathbf{h}_t^{(\mathrm{A})} = (\mathbf{s}_1^{(\mathrm{A})}, \mathbf{a}_1^{(\mathrm{A})}, \mathbf{o}_1, \dots, \mathbf{a}_{t-1}^{(\mathrm{A})}, \mathbf{s}_t^{(\mathrm{A})}, \mathbf{o}_t) \in \mathcal{H}_{\mathrm{A}}$$

Objective: (i) disrupt workflows; and (ii) compromise nodes:

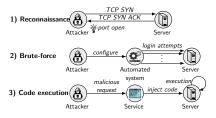


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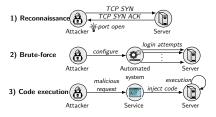


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Objective: (i) disrupt workflows; and (ii) compromise nodes:



The Intrusion Response Problem

$$\mathbf{s}_{t+1}^{(\mathrm{A})} \sim f_{\mathrm{A}}(\cdot \mid \mathbf{S}_{t}^{(\mathrm{A})}, \mathbf{A}_{t}) \qquad \forall t$$

$$\mathbf{o}_{t+1} \sim Z(\cdot \mid \mathbf{S}_{t+1}^{(\mathrm{D})}, \mathbf{A}_{t}^{(\mathrm{A})}) \qquad \forall t$$

$$\mathbf{a}_t^{(\mathrm{A})} \sim \pi_{\mathrm{A}}(\cdot \mid \mathbf{H}_t^{(\mathrm{A})}), \ \mathbf{a}_t^{(\mathrm{A})} \in \mathcal{A}_{\mathrm{A}}(\mathbf{s}_t) \qquad orall t$$

$$\mathbf{a}_t^{(\mathrm{D})} \sim \pi_{\mathrm{D}}(\cdot \mid \mathbf{H}_t^{(\mathrm{D})}), \ \mathbf{a}_t^{(\mathrm{D})} \in \mathcal{A}_{\mathrm{D}} \qquad \forall t$$

 $\mathbb{E}_{(\pi_{\mathrm{D}},\pi_{\mathrm{A}})}$ denotes the expectation of the random vectors $(\mathbf{S}_t, \mathbf{O}_t, \mathbf{A}_t)_{t \in \{1, \dots, T\}}$ when following the strategy profile $(\pi_{\mathrm{D}}, \pi_{\mathrm{A}})$

(1) can be formulated as a zero-sum Partially Observed Stochastic Game with Public Observations (a PO-POSG):

$$\Gamma = \langle \mathcal{N}, (\mathcal{S}_i)_{i \in \mathcal{N}}, (\mathcal{A}_i)_{i \in \mathcal{N}}, (f_i)_{i \in \mathcal{N}}, u, \gamma, (\mathbf{b}_1^{(i)})_{i \in \mathcal{N}}, \mathcal{O}, Z \rangle$$

Existence of a Solution

Theorem

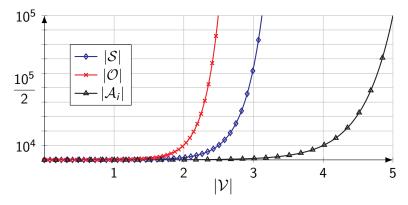
Given the <code>PO-POSG</code> Γ , the following holds:

(A) Γ has a mixed Nash equilibrium and a value function $V^* : \mathcal{B}_D \times \mathcal{B}_A \to \mathbb{R}.$

(B) For each strategy pair $(\pi_A, \pi_D) \in \Pi_A \times \Pi_D$, the best response sets $B_D(\pi_A)$ and $B_A(\pi_D)$ are non-empty.

The Curse of Dimensionality

While Γ has a value, computing it is intractable. The state, action, and observation spaces of the game grow exponentially with |V|.



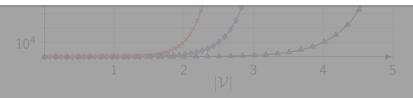
Growth of |S|, |O|, and $|A_i|$ in function of the number of nodes |V|

The Curse of Dimensionality

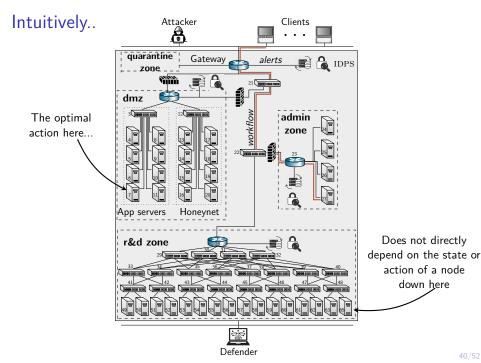
While (1) has a solution (i.e the game Γ has a value (Thm 1)), computing it is intractable since the state, action, and observation spaces of the game grow exponentially with |V|.

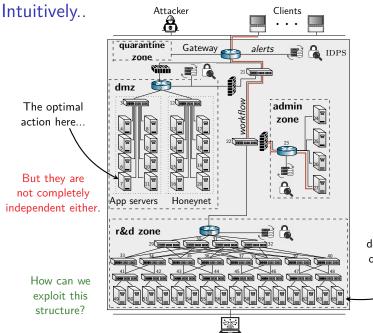


We tackle the scability challenge with decomposition



Growth of |S|, |O|, and $|A_i|$ in function of the number of nodes |V|





Defender

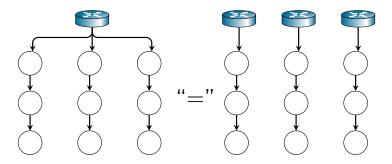
Does not directly depend on the state or action of a node down here

Our Approach: System Decomposition

To avoid explicitly enumerating the very large state, observation, and action spaces of Γ , we exploit three structural properties.

- 1. Additive structure across workflows.
 - The game decomposes into additive subgames on the workflow-level
- 2. Optimal substructure within a workflow.
 - The subgame for each workflow decomposes into subgames on the node-level with optimal substructure
- 3. Threshold properties of local defender strategies.
 - Optimal node-level strategies exhibit threshold structures

Additive Structure Across Workflows (Intuition)



- If there is no path between i and j in G, then i and j are independent in the following sense:
 - Compromising i has no affect on the state of j.
 - Compromising i does not make it harder or easier to compromise j.

Compromising i does not affect the service provided by j.

- Defending i does not affect the state of j.
- Defending i does not affect the service provided by j.

Additive Structure Across Workflows

Definition (Transition independence)

A set of nodes ${\mathcal Q}$ are transition independent iff the transition probabilities factorize as

$$f(\mathbf{S}_{t+1} \mid \mathbf{S}_t, \mathbf{A}_t) = \prod_{i \in \mathcal{Q}} f(\mathbf{S}_{t+1,i} \mid \mathbf{S}_{t,i}, \mathbf{A}_{t,i})$$

Definition (Utility independence)

A set of nodes Q are utility independent iff there exists functions $u_1, \ldots, u_{|Q|}$ such that the utility function u decomposes as

$$u(\mathbf{S}_t, \mathbf{A}_t) = f(u_1(\mathbf{S}_{t,1}, \mathbf{A}_{t,1}), \dots, u_1(\mathbf{S}_{t,|\mathcal{Q}|}, \mathbf{A}_{t,\mathcal{Q}}))$$

and

$$u_i \leq u'_i \iff f(u_1,\ldots,u_i,\ldots,u_{|\mathcal{Q}|}) \leq f(u_1,\ldots,u'_i,\ldots,u_{|\mathcal{Q}|})$$

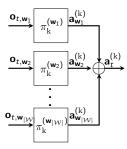
Additive Structure Across Workflows

Theorem (Node independencies)

(A) All nodes \mathcal{V} in the game Γ are transition independent. (B) If there is no path between i and j in the topology graph \mathcal{G} , then i and j are utility independent.

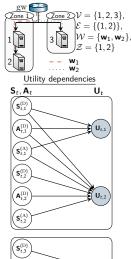
Corollary (Additive structure across workflows)

 Γ decomposes into $|\mathcal{W}|$ additive subproblems that can be solved independently and in parallel.



Optimal Substructure Within a Workflow IT infrastructure

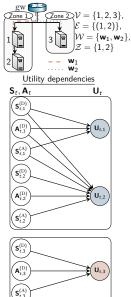
- Nodes in the same workflow are utility dependent.
- Adding locally-optimal strategies <u>does not</u> yield an optimal workflow strategy.
- However, the locally-optimal strategies satisfy the optimal substructure property.
- ⇒ there exists an algorithm for constructing an optimal workflow strategy from locally-optimal strategies for each node.



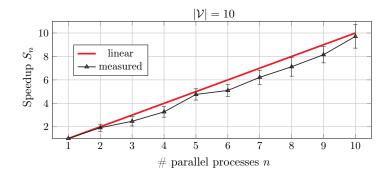
A(D)

Optimal Substructure Within a Workflow IT infrastructure

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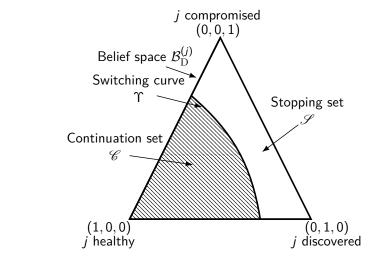


Scalable Learning through Decomposition



Speedup of best response computation for the decomposed game; T_n denotes the completion time with *n* processes; the speedup is calculated as $S_n = \frac{T_1}{T_n}$; the error bars indicate standard deviations from 3 measurements.

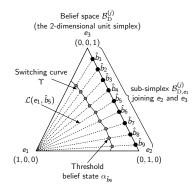
Threshold Properties of Local Defender Strategies.



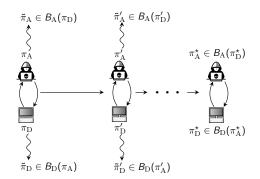
- A node can be in three attack states s_t^(A): Healthy, Discovered, Compromised.
- The defender has a belief state $\mathbf{b}_t^{(D)}$

Proof Sketch (Threshold Properties)

- Let L(e₁, b) denote the line segment that starts at the belief state
 e₁ = (1,0,0) and ends at b, where b is in the sub-simplex that joins e₂ and e₃.
- All beliefs on L(e₁, b̂) are totally ordered according to the Monotone Likelihood Ratio (MLR) order. ⇒ a threshold belief state α_{b̂} ∈ L(e₁, b̂) exists where the optimal strategy switches from C to S.
- Since the entire belief space can be covered by the union of lines *L*(*e*₁, *b̂*), the threshold belief states *α*_{*b*₁}, *α*_{*b*₂},... yield a switching curve Υ.



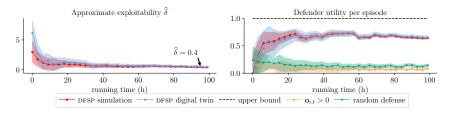
Decompositional Fictitious Play (DFSP)



Fictitious play: iterative averaging of best responses.

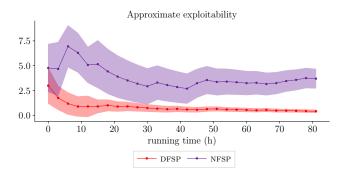
- Learn best response strategies iteratively through the parallel solving of subgames in the decomposition
- Average best responses to approximate the equilibrium

Learning Equilibrium Strategies



Learning curves obtained during training of DFSP to find optimal (equilibrium) strategies in the intrusion response game; **red and blue curves relate to dfsp**; black, orange and green curves relate to baselines.

Comparison with NFSP



Learning curves obtained during training of DFSP and NFSP to find optimal (equilibrium) strategies in the intrusion response game; **the red curve relate to dfsp** and the purple curve relate to NFSP; all curves show simulation results.

Conclusions

- We develop a *framework* to automatically learn security strategies.
- We apply the framework to an intrusion response use case.
- We derive properties of optimal security strategies.
- We evaluate strategies on a digital twin.
- Questions \rightarrow demonstration

