Learning Intrusion Prevention Policies Through Optimal Stopping

NSE Seminar

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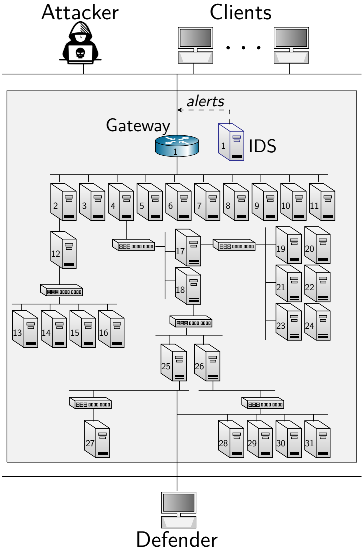
Use Case: Intrusion Prevention

- A **Defender** owns an infrastructure
  - Consists of connected components
  - Components run network services
  - Defender defends the infrastructure by monitoring and active defense
- An **Attacker** seeks to intrude on the infrastructure
  - Has a partial view of the infrastructure
  - Wants to compromise specific components
  - Attacks by reconnaissance, exploitation and pivoting



Use Case: Intrusion Prevention

- A **Defender** owns an infrastructure
  - Consists of connected components
  - Components run network services
  - Defender defends the infrastructure

We formulate this use case as an **Optimal Stopping** problem

infrastructure

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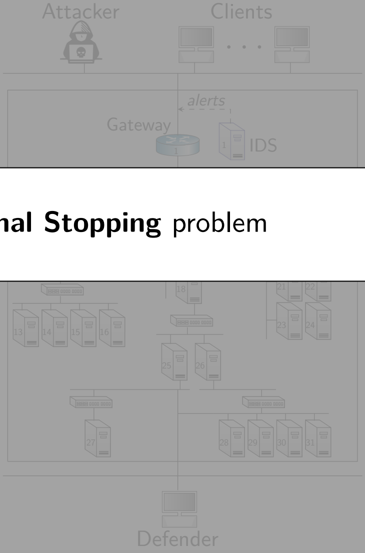

Background on Optimal Stopping Problems

- **The General Problem**:
  - A Markov process $(s_t)_{t=1}^{T}$ is observed sequentially
  - Two options per t: (i) continue to observe; or (ii) stop
  - Find the *optimal stopping time* τ^*:

$$\tau^* = \arg\max_{\tau} \mathbb{E}_{\tau}\left[\sum_{t=1}^{\tau-1}\gamma^{t-1}\mathcal{R}^{C}_{s_t s_{t+1}} + \gamma^{\tau-1}\mathcal{R}^{S}_{s_\tau s_\tau}\right] \qquad (1)$$

  where $\mathcal{R}^{S}_{ss'}$ & $\mathcal{R}^{C}_{ss'}$ are the stop/continue rewards

- **History**:
  - Studied in the 18th century to analyze a gambler's fortune
  - Formalized by Abraham Wald in 1947
  - Since then it has been generalized and developed by (Chow, Shiryaev & Kolmogorov, Bather, Bertsekas, etc.)

- **Applications & Use Cases**:
  - Change detection, machine replacement, hypothesis testing, gambling, selling decisions, queue management, advertisement scheduling, the secretary problem, etc.

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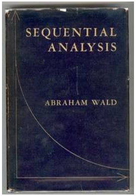

[1] Abraham Wald. *Sequential Analysis*. Wiley and Sons, New York, 1947.

[2] Y. Chow, H. Robbins, and D. Siegmund. "Great expectations: The theory of optimal stopping". In: 1971.

[3] Albert N. Shirayev. *Optimal Stopping Rules*. Reprint of russian edition from 1969. Springer-Verlag Berlin, 2007.

[4] John Bather. *Decision Theory: An Introduction to Dynamic Programming and Sequential Decisions*. USA: John Wiley and Sons, Inc., 2000. ISBN: 0471976490.

[5] Dimitri P. Bertsekas. *Dynamic Programming and Optimal Control*. 3rd. Vol. I. Belmont, MA, USA: Athena

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- **Applications & Use Cases**:
  - Change detection[6], selling decisions[7], queue management[8], advertisement scheduling[9], etc.

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[14] Kim Hammar and Rolf Stadler. *Learning Intrusion Prevention Policies through Optimal Stopping*. 2021. arXiv: 2106.07160 [cs.AI].

Optimal Stopping: The General Theory

- Two general approaches: the *Markovian approach* and the *martingale approach*.

- **The Markovian approach**:
  - Model the problem as a MDP or POMDP
  - A policy π^* that satisfies the Bellman-Wald equation is optimal:

$$\pi^*(s) = \arg\max_{\{S,C\}} \left[\underbrace{\mathbb{E}\left[\mathcal{R}_s^S\right]}_{\text{stop}}, \underbrace{\mathbb{E}\left[\mathcal{R}_s^C + \gamma V^*(s')\right]}_{\text{continue}} \right] \quad \forall s \in \mathcal{S}$$

  - Solve by backward induction, dynamic programming, or reinforcement learning

- **The martingale approach**:
  - Model the state process as an arbitrary stochastic process
  - The reward of the optimal stopping time is given by the *smallest supermartingale that stochastically dominates the process*, called the Snell envelope [14].

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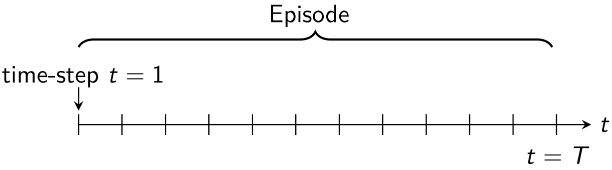
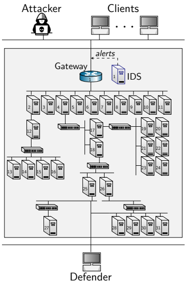
[15] J. L. Snell. "Applications of martingale system theorems". In: *Transactions of the American Mathematical Society* 73 (1952), pp. 293–312.

Formulating Intrusion Prevention as a Stopping Problem





- **Intrusion Prevention as Optimal Stopping Problem**:
  - The system evolves in discrete time-steps.
  - Defender observes the infrastructure (IDS, log files, etc.).
  - An intrusion occurs at an unknown time.
  - The defender can make L stops.
  - Each stop is associated with a defensive action
  - The final stop shuts down the infrastructure.
  - **Based on the observations, when is it optimal to stop?**
  - We formalize this problem with a POMDP

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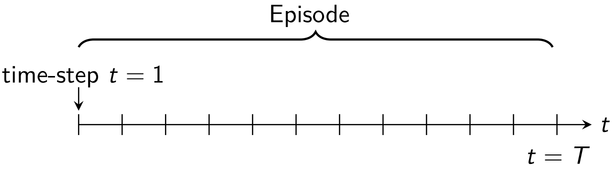

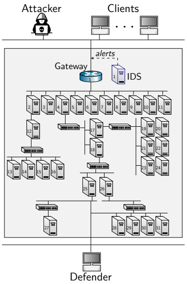

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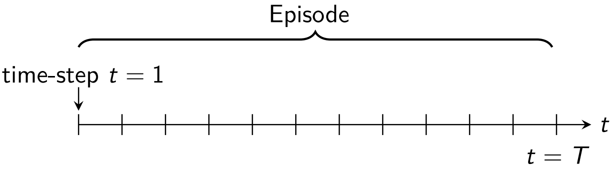

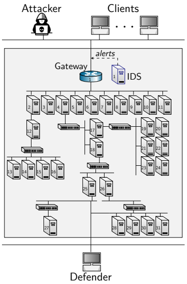

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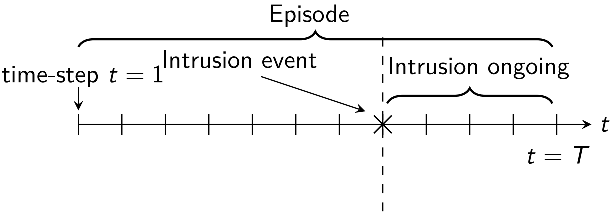

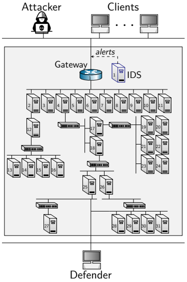

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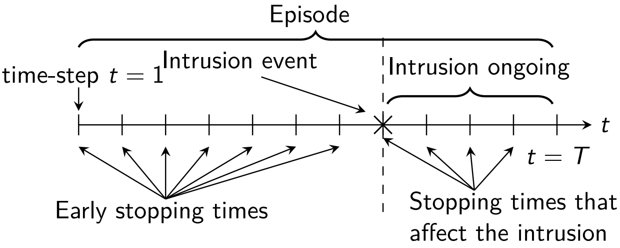

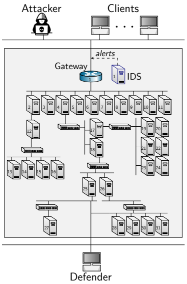

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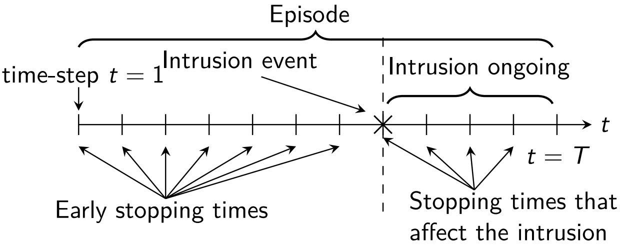

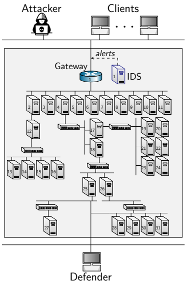

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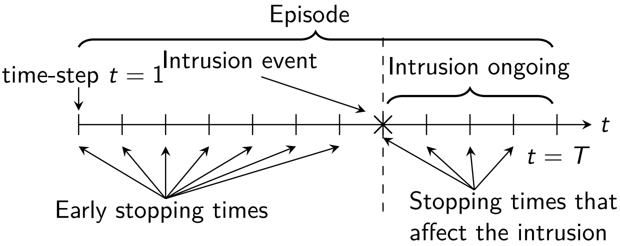

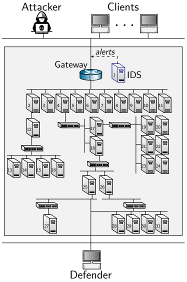

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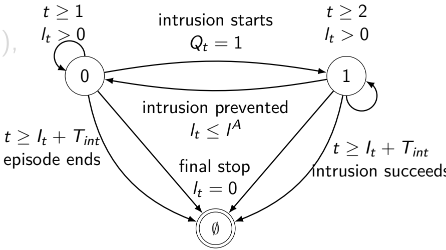
A Partially Observed MDP Model for the Defender

- **States:**
  - Intrusion state $s_t \in \{0, 1\}$, terminal $\emptyset$.
- **Observations:**
  - Severe/Warning IDS Alerts $(\Delta x, \Delta y)$, Login attempts Δz, stops remaining $l_t \in \{1, .., L\}$, $f_{XYZ}(\Delta x, \Delta y, \Delta z | s_t, l_t, t)$
- **Actions:**
  - "Stop" (S) and "Continue" (C)
- **Rewards:**
  - Reward: security and service. Penalty: false alarms and intrusions
- **Transition probabilities:**
  - Bernoulli process $(Q_t)_{t=1}^{T} \sim Ber(p)$ defines intrusion start $I_t \sim Ge(p)$
- **Objective and Horizon:**
  - $\max \mathbb{E}_{\pi_\theta}\left[\sum_{t=1}^{T_\emptyset} r(s_t, a_t)\right]$, $T_\emptyset$



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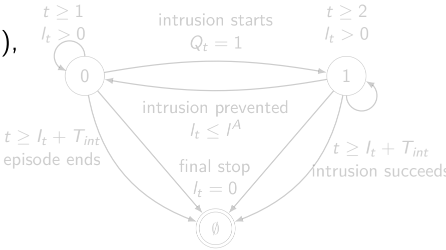

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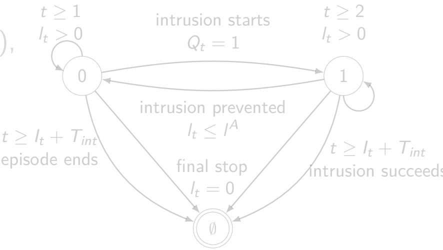

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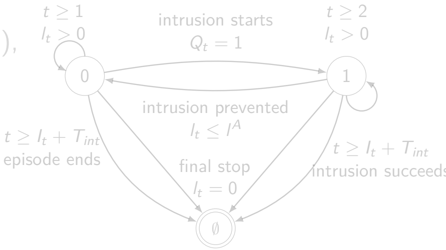

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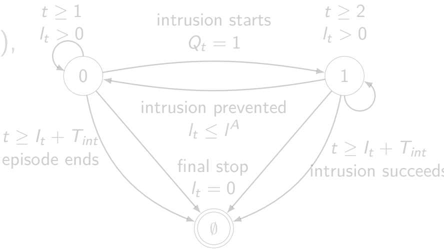

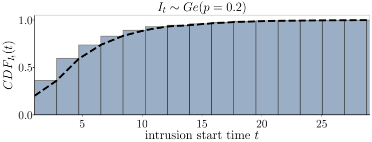

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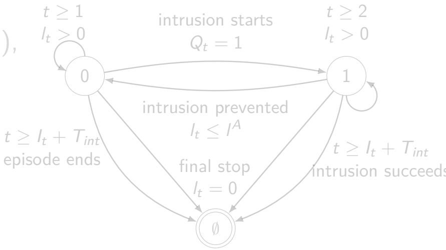

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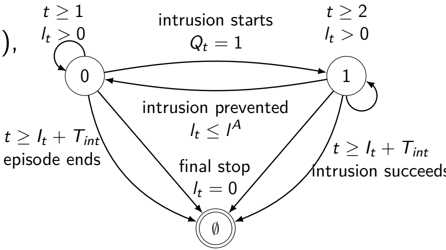

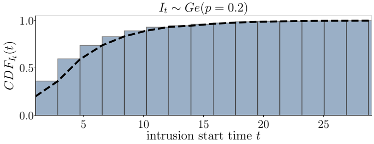

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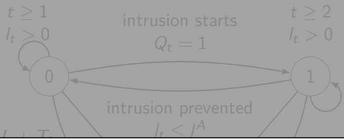
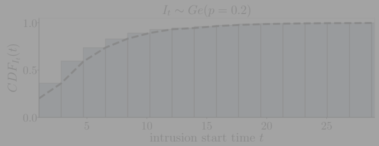
- **Observations:**
  - Severe/Warning IDS Alerts $(\Delta x, \Delta y)$, Login attempts Δz, stops remaining $l_t \in \{1, .., L\}$,



We analyze the optimal policy using optimal stopping theory

- **Rewards:**
  - Reward: security and service. Penalty: false alarms and intrusions
- **Transition probabilities:**
  - Bernoulli process $(Q_t)_{t=1}^{T} \sim Ber(p)$ defines intrusion start $I_t \sim Ge(p)$
- **Objective and Horizon:**
  - $\max \mathbb{E}_{\pi_\theta}\left[\sum_{t=1}^{T_\emptyset} r(s_t, a_t)\right]$, $T_\emptyset$



Threshold Properties of the Optimal Defender Policy (1/4)

- **Belief States:**
  - The belief state $b_t \in \mathcal{B}$ is defined as $b_t(s_t) = \mathbb{P}[s_t | h_t]$
  - b_t is a sufficient statistic of s_t based on $h_t = (\rho_1, a_1, o_1, \ldots, a_{t-1}, o_t) \in \mathcal{H}$
  - $\mathcal{B}$ is the unit $(|\mathcal{S}| - 1)$-simplex

- **Characterizing the Optimal Policy π^*:**
  - To characterize the optimal policy π^* we partition $\mathcal{B}$ based on optimal actions and stops remaining l.
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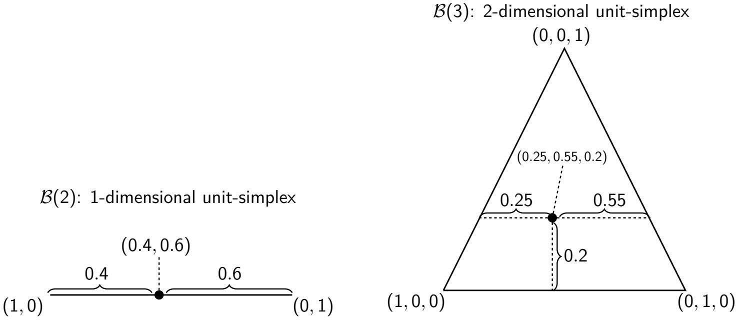

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Threshold Properties of the Optimal Defender Policy (2/4)

Theorem

1. $\mathscr{S}^{l-1} \subseteq \mathscr{S}^{l}$ *for* $l = 2, \ldots L$.
2. *If* $L = 1$, *there exists an optimal threshold* $\alpha^* \in [0, 1]$ *and an optimal policy of the form:*

$$\pi^*(b(1)) = S \iff b(1) \geq \alpha^* \tag{2}$$

3. *If* $L \geq 1$ *and* f_{XYZ}, *is totally positive of order 2 (TP2), there exists* L *optimal thresholds* $\alpha_l^* \in [0, 1]$ *and an optimal policy of the form:*

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where α_l^* *is decreasing in* l.

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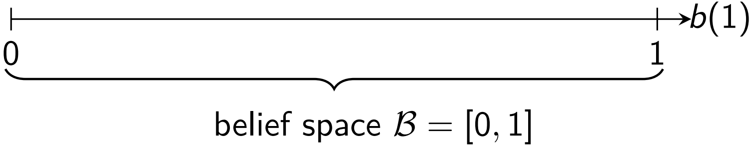
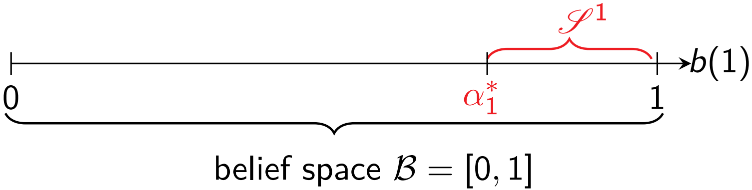


Threshold Properties of the Optimal Defender Policy (3/4)



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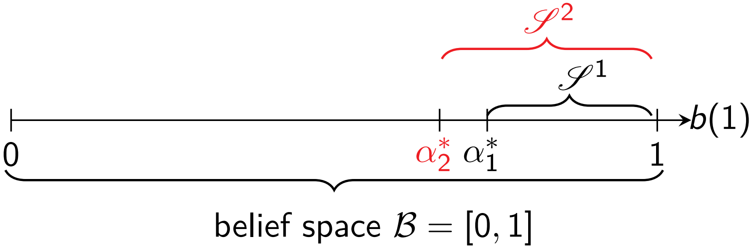

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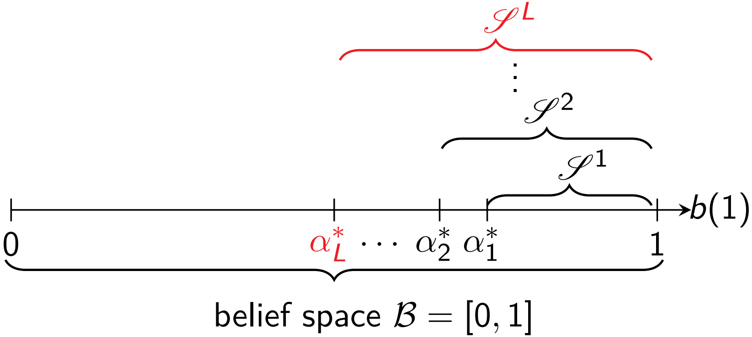

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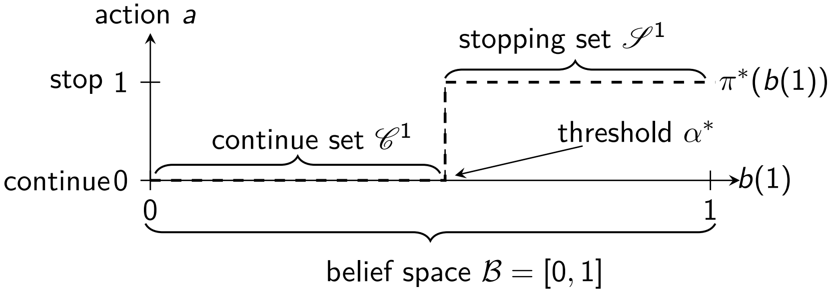

Threshold Properties of the Optimal Defender Policy (4/4)

- **What are the implications of these results about π^*?**
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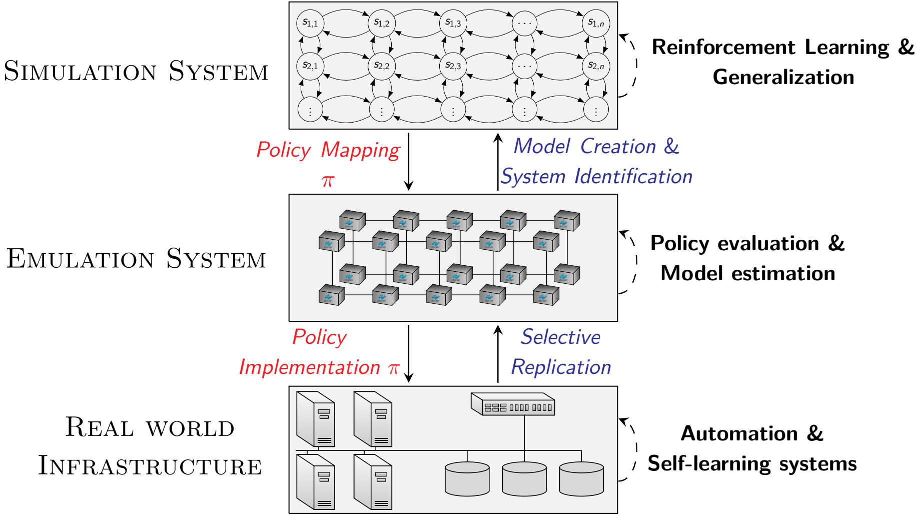
Our Method for Finding Effective Security Strategies



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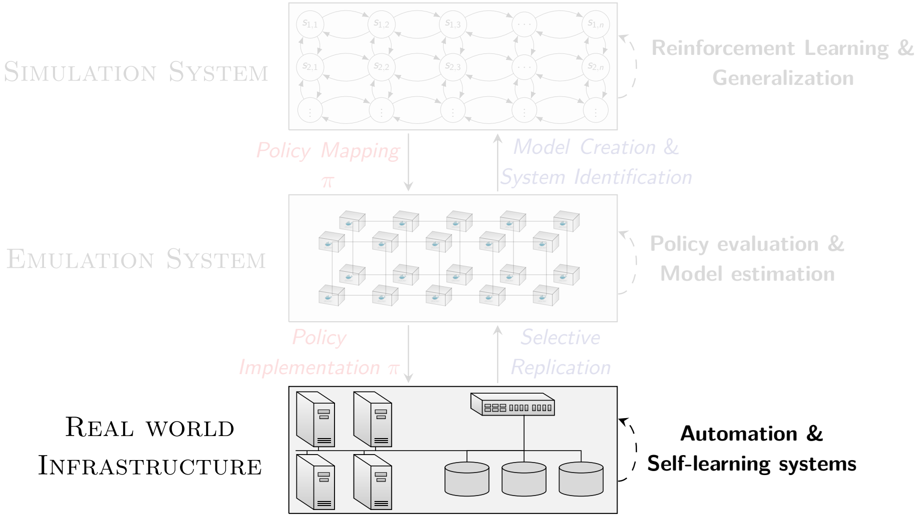

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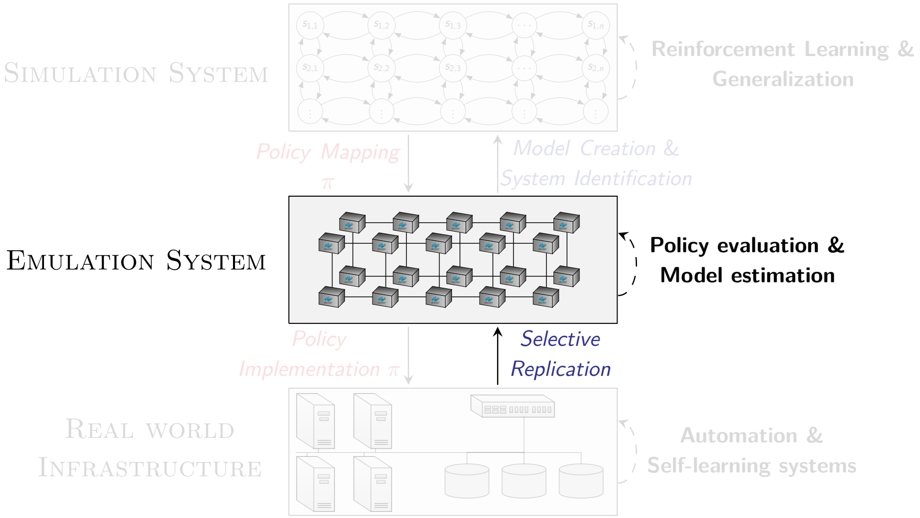

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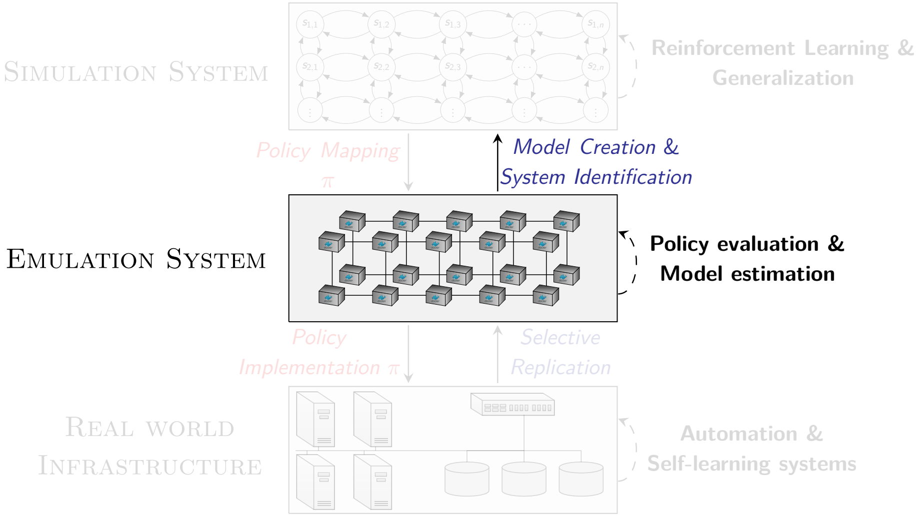

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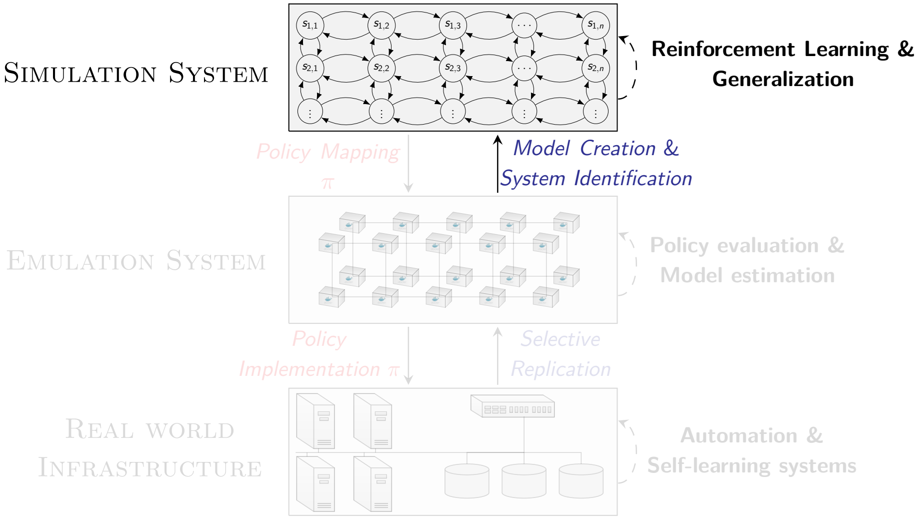

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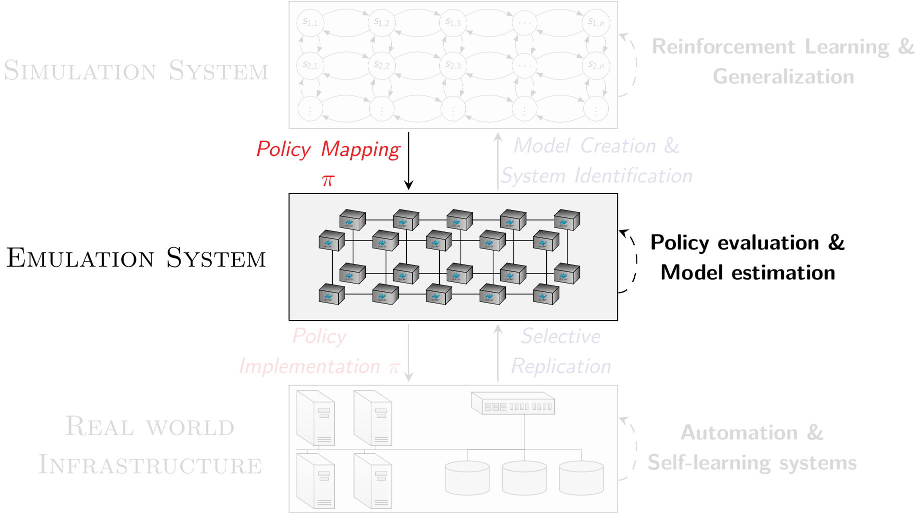

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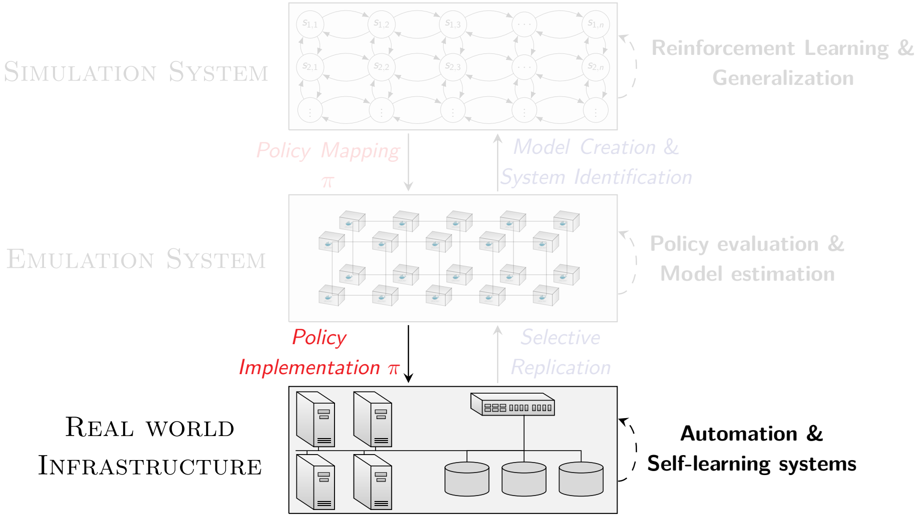

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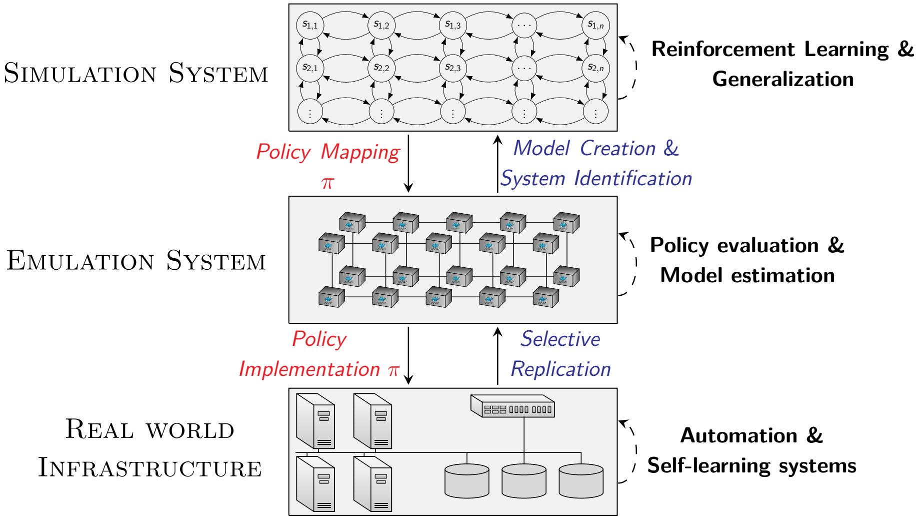

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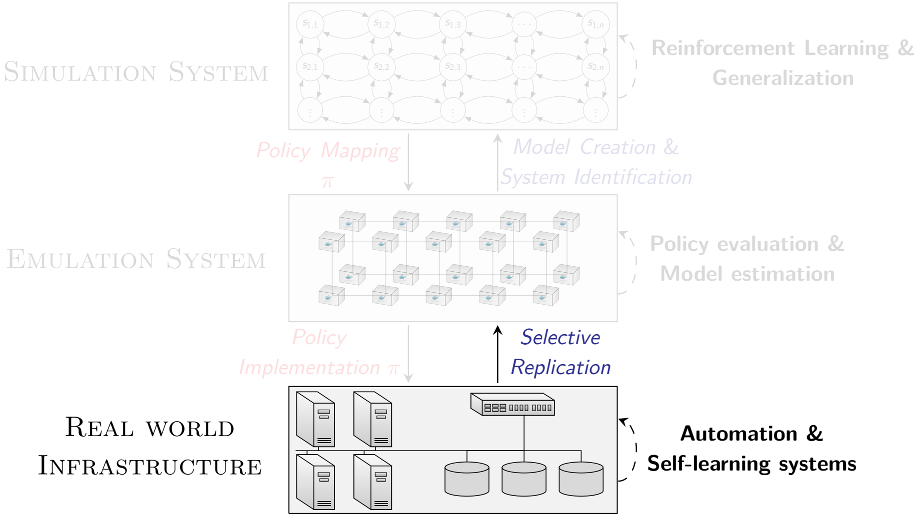
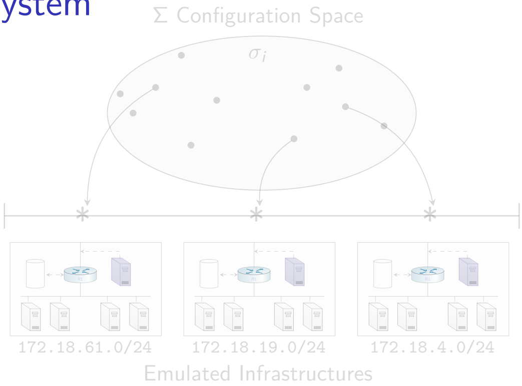


Emulation System



Emulation

A cluster of machines that runs a virtualized infrastructure which replicates important functionality of target systems.

- The set of virtualized configurations define a *configuration space* $\Sigma = \langle \mathcal{A}, \mathcal{O}, \mathcal{S}, \mathcal{U}, \mathcal{T}, \mathcal{V} \rangle$.
- A specific emulation is based on a configuration $\sigma_i \in \Sigma$.

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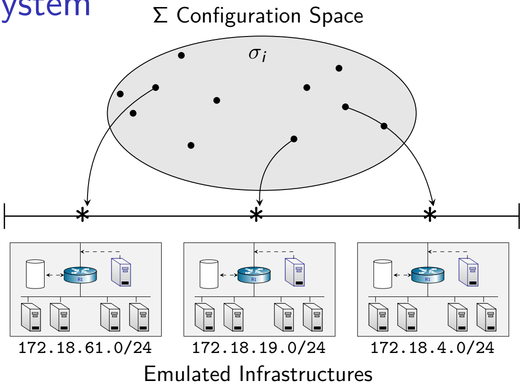

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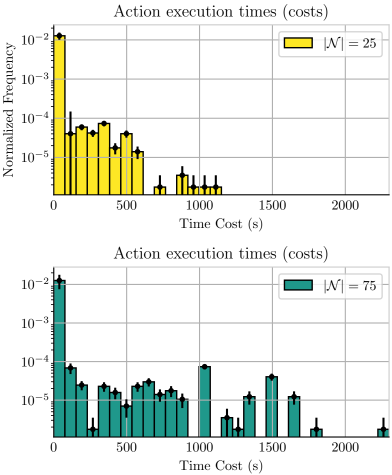
Emulation: Execution Times of Replicated Operations



- **Fundamental issue**: Computational methods for policy learning typically require samples on the order of $100k - 10M$.
- $\implies$ Infeasible to optimize in the emulation system

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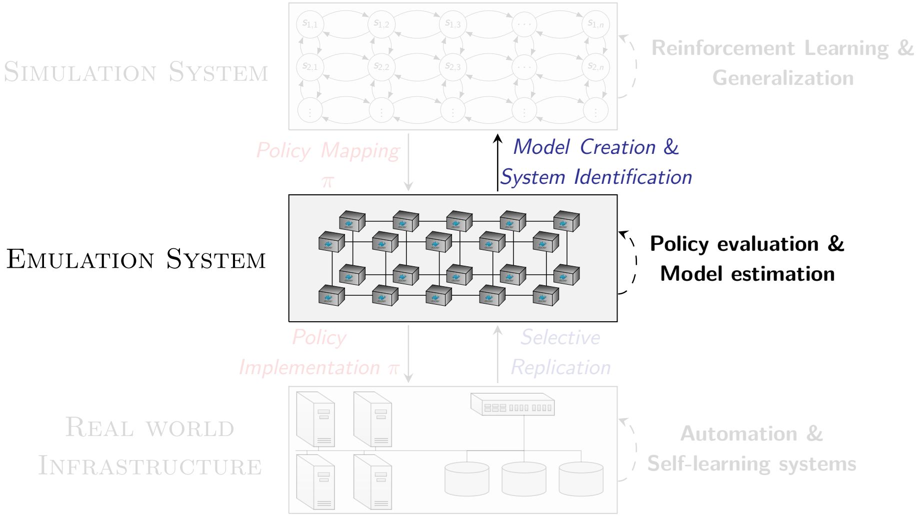
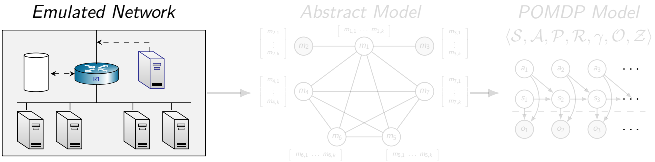


From Emulation to Simulation: System Identification



- **Abstract Model Based on Domain Knowledge**: Models the set of *controls*, the *objective function*, and the *features* of the emulated network.
  - Defines the static parts a POMDP model.

- **Dynamics Model ($\mathcal{P}$, $\mathcal{Z}$) Identified using System Identification**: Algorithm based on random walks and maximum-likelihood estimation.

$$\mathcal{M}(b'|b,a) \triangleq \frac{n(b,a,b')}{\sum_{j'} n(s,a,j')}$$

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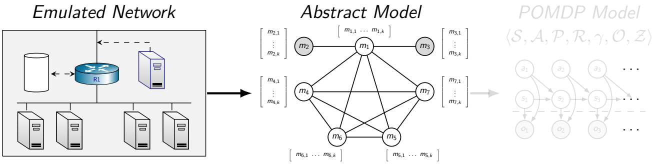

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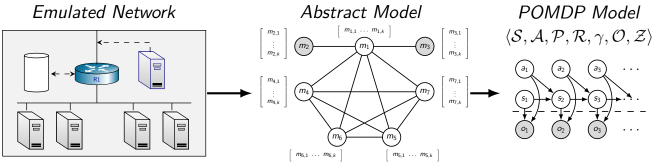

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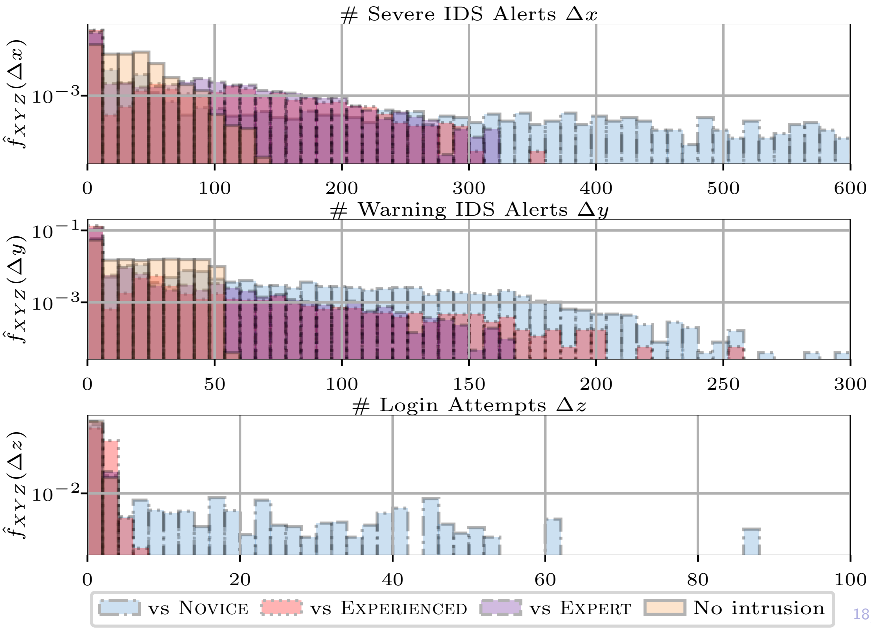
System Identification: Estimated Empirical Distributions



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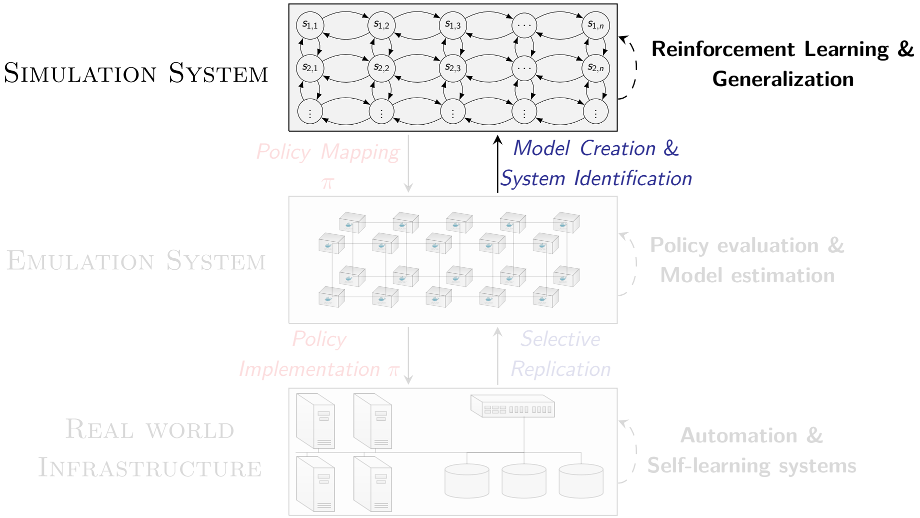

Policy Optimization in the Simulation System using Reinforcement Learning

- **Goal**:
  - Approximate $\pi^* = \arg\max_\pi \mathbb{E}\left[\sum_{t=1}^{T} \gamma^{t-1} r_{t+1}\right]$

- **Learning Algorithm**:
  - Represent π by π_θ
  - Define objective $J(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=1}^{T} \gamma^{t-1} r(s_t, a_t)\right]$
  - Maximize $J(\theta)$ by stochastic gradient ascent

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\underbrace{\nabla_\theta \log \pi_\theta(a|h)}_{\text{actor}} \underbrace{A^{\pi_\theta}(h, a)}_{\text{critic}}\right]$$

- **Method**:
  1. Simulate a series of POMDP episodes
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- **Method**:
  1. Simulate a series of POMDP episodes
  2. Use episode outcomes and trajectories to estimate $\nabla_\theta J(\theta)$
  3. Update policy π_θ with stochastic gradient ascent
  4. Continue until convergence

Policy Optimization in the Simulation System using Reinforcement Learning

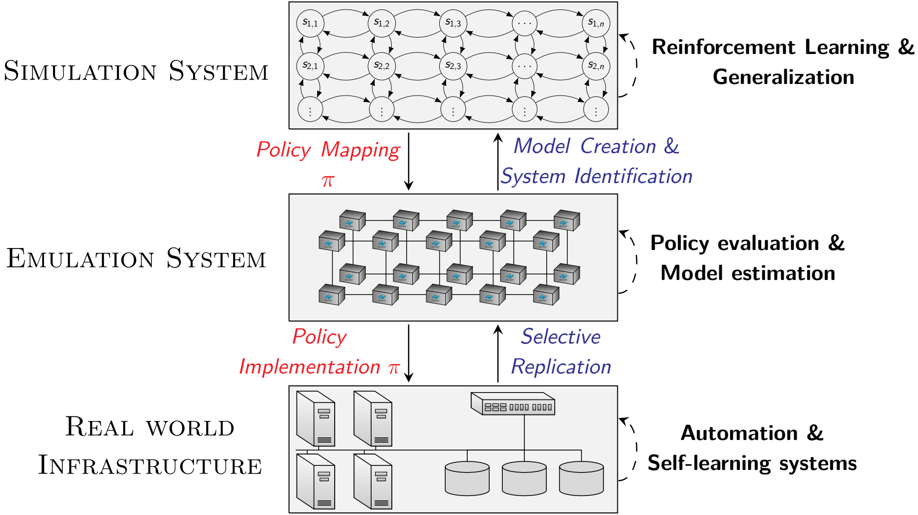
- **Goal**:
  - Approximate $\pi^* = \arg\max_\pi \mathbb{E}[\sum_{t=1}^{T} \gamma^{t-1} r_{t+1}]$
- **Learning Algorithm**:
  - Represent π by π_θ
  - Define objective $J(\theta) = \mathbb{E}_{\pi_\theta}[\sum_{t=1}^{T} \gamma^{t-1} r(s_t, a_t)]$
  - Maximize $J(\theta)$ by stochastic gradient ascent
    $\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(a|h) A^{\pi_\theta}(h, a)]$
- **Method**:
  1. Simulate a series of POMDP episodes
  2. Use episode outcomes and trajectories to estimate $\nabla_\theta J(\theta)$
  3. Update policy π_θ with stochastic gradient ascent
  4. Continue until convergence

- *Finding Effective Security Strategies through Reinforcement Learning and Self-Play*[a]
- *Learning Intrusion Prevention Policies through Optimal Stopping*[b]

[a] Kim Hammar and Rolf Stadler. "Finding Effective Security Strategies through Reinforcement Learning and Self-Play". In: *International Conference on Network and Service Management (CNSM)*. Izmir, Turkey, Nov. 2020.

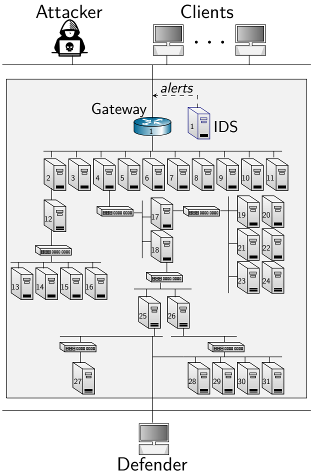
[b] Kim Hammar and Rolf Stadler. *Learning Intrusion Prevention Policies through Optimal Stopping*. 2021. arXiv: `2106.07160 [cs.AI]`.

Our Method for Finding Effective Security Strategies



The Target Infrastructure

- **Topology**:
  - 30 Application Servers, 1 Gateway/IDS (Snort), 3 Clients, 1 Attacker, 1 Defender
- **Services**
  - 31 SSH, 8 HTTP, 1 DNS, 1 Telnet, 2 FTP, 1 MongoDB, 2 SMTP, 2 Teamspeak 3, 22 SNMP, 12 IRC, 1 Elasticsearch, 12 NTP, 1 Samba, 19 PostgreSQL
- **RCE Vulnerabilities**
  - 1 CVE-2010-0426, 1 CVE-2014-6271, 1 SQL Injection, 1 CVE-2015-3306, 1 CVE-2016-10033, 1 CVE-2015-5602, 1 CVE-2015-1427, 1 CVE-2017-7494
  - 5 Brute-force vulnerabilities
- **Operating Systems**
  - 23 Ubuntu-20, 1 Debian 9:2, 1 Debian Wheezy, 6 Debian Jessie, 1 Kali



Target infrastructure.

Emulating the Client Population

Client	*Functions*	*Application servers*
1	HTTP, SSH, SNMP, ICMP	N_2, N_3, N_{10}, N_{12}
2	IRC, PostgreSQL, SNMP	$N_{31}, N_{13}, N_{14}, N_{15}, N_{16}$
3	FTP, DNS, Telnet	N_{10}, N_{22}, N_4

Table 1: Emulated client population; each client interacts with application servers using a set of functions at short intervals.

Emulating the Defender's Actions

l_t	*Action*	*Command in the Emulation*
3	Reset users	`deluser -remove-home <username>`
2	Blacklist IPs	`iptables -A INPUT -s <ip> -j DROP`
1	Block gateway	`iptables -A INPUT -i eth0 -j DROP`

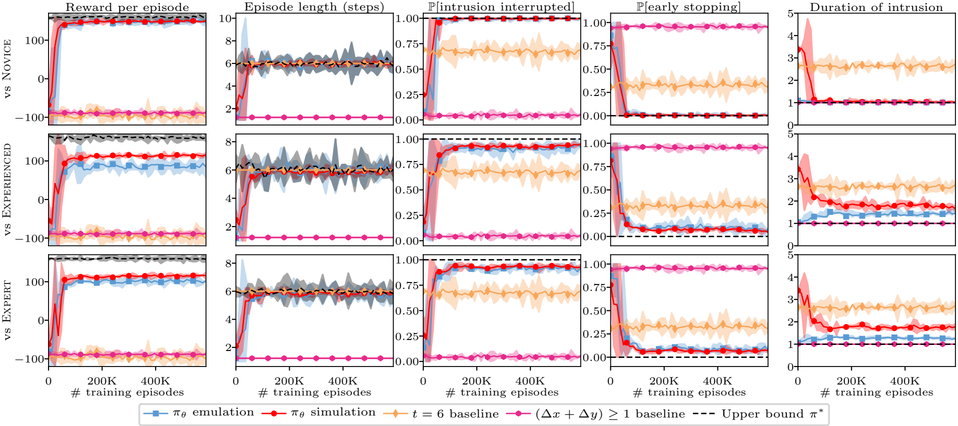
Table 2: Commands used to implement the defender's stop actions in the emulation.

Static Attackers to Emulate Intrusions

Time-steps t	NoviceAttacker	ExperiencedAttacker	ExpertAttacker
1-$I_t \sim Ge(0.2)$	(Intrusion has not started)	(Intrusion has not started)	(Intrusion has not started)
$I_t + 1$-$I_t + 6$	Recon$_1$, brute-force attacks (SSH,Telnet,FTP) on N_2, N_4, N_{10}, login(N_2, N_4, N_{10}), backdoor(N_2, N_4, N_{10})	Recon$_2$, CVE-2017-7494 exploit on N_4, brute-force attack (SSH) on N_2, login(N_2, N_4), backdoor(N_2, N_4), Recon$_2$	Recon$_3$, CVE-2017-7494 exploit on N_4, login(N_4), backdoor(N_4) Recon$_3$, SQL Injection on N_{18}
$I_t + 7$-$I_t + 10$	Recon$_1$, CVE-2014-6271 on N_{17}, login(N_{17}), backdoor(N_{17})	CVE-2014-6271 on N_{17}, login(N_{17}) backdoor(N_{17}), SSH brute-force attack on N_{12}	login(N_{18}), backdoor(N_{18}), Recon$_3$, CVE-2015-1427 on N_{25}
$I_t + 11$-$I_t + 14$	SSH brute-force attack on N_{12}, login(N_{12}) CVE-2010-0426 exploit on N_{12}, Recon$_1$	login(N_{12}), CVE-2010-0426 exploit on N_{12}, Recon$_2$, SQL Injection on N_{18}	login(N_{25}), backdoor(N_{25}), Recon$_3$, CVE-2017-7494 exploit on N_{27}
$I_t + 15$-$I_t + 16$		login(N_{18}), backdoor(N_{18})	login(N_{27}), backdoor(N_{27})
$I_t + 17$-$I_t + 19$		Recon$_2$, CVE-2015-1427 on N_{25}, login(N_{25})	

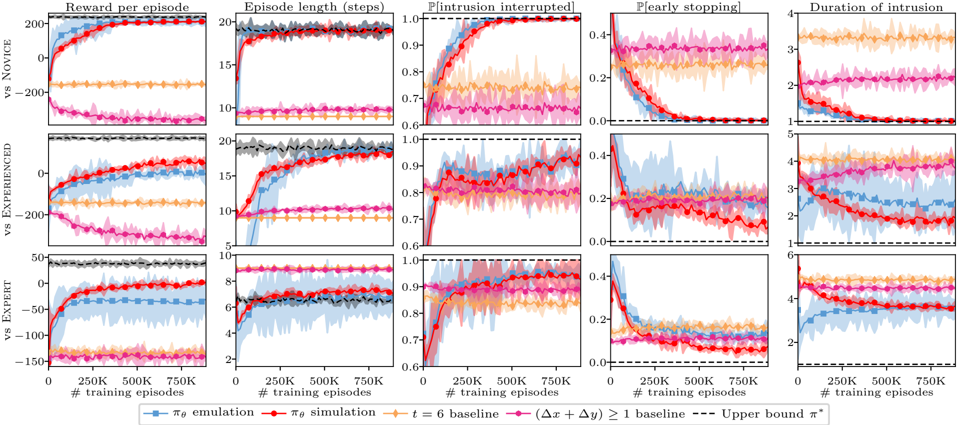
Table 3: Attacker actions to emulate intrusions.

Learning Intrusion Prevention Policies through Optimal Stopping



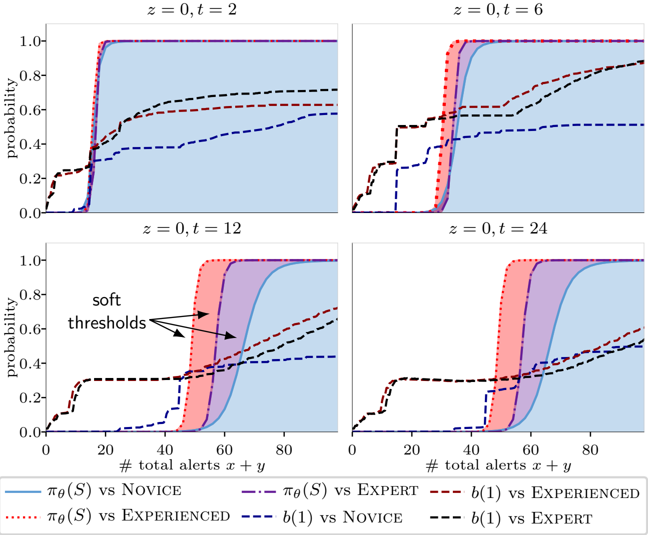
Learning curves of training defender policies against static attackers, $L = 1$.

Learning Intrusion Prevention Policies through Optimal Stopping

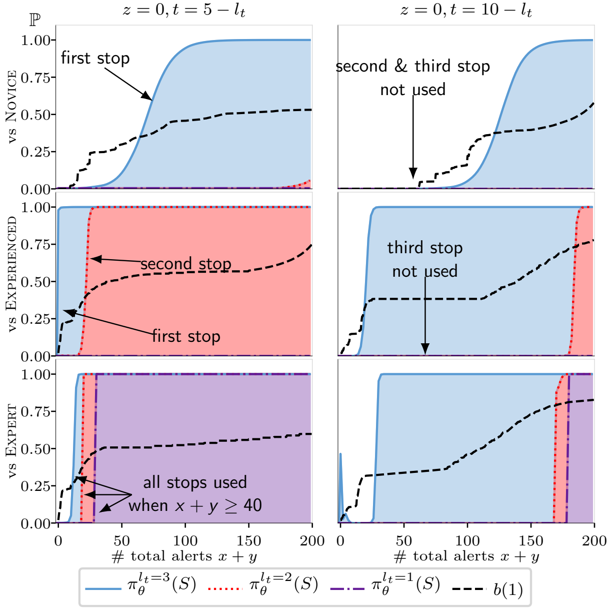


Learning curves of training defender policies against static attackers, $L = 3$.

Threshold Properties of the Learned Policies, $L = 1$



Threshold Properties of the Learned Policies, $L = 3$



Conclusions & Future Work

- **Conclusions:**
  - We develop a *method* to find learn intrusion prevention policies
    - (1) emulation system; (2) system identification; (3) simulation system; (4) reinforcement learning and (5) domain randomization and generalization.
  - We formulate intrusion prevention as a multiple stopping problem
    - We present a POMDP model of the use case
    - We apply the stopping theory to establish structural results of the optimal policy
- **Our research plans:**
  - Extending the theoretical model
    - Relaxing simplifying assumptions (e.g. more dynamic defender actions)
    - Active attacker
  - Evaluation on real world infrastructures