Intrusion Prevention through Optimal Stopping

Invited Talk @Alan Turing Institute London

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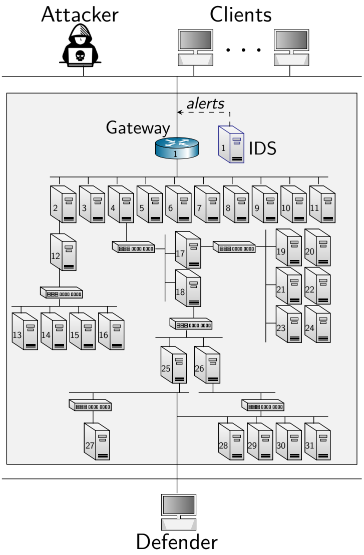
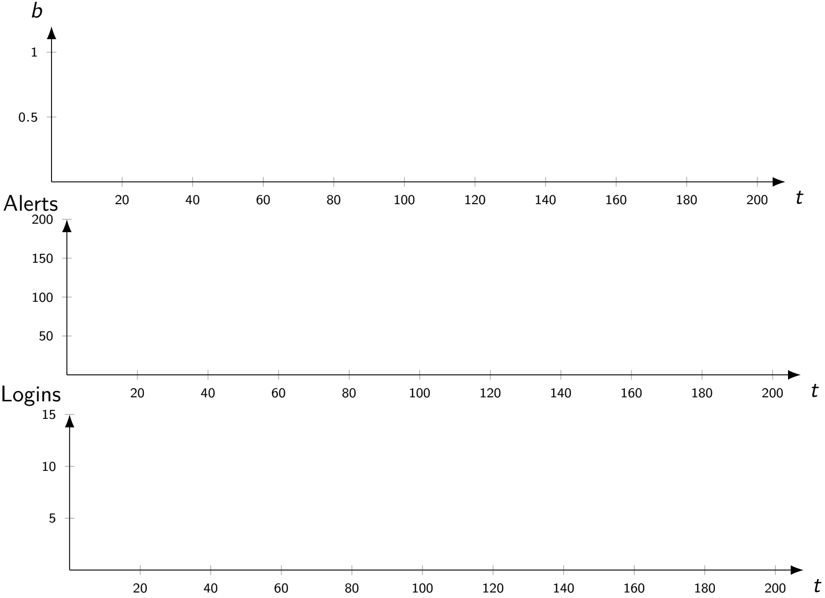
Mar 25, 2022

Use Case: Intrusion Prevention

- A **Defender** owns an infrastructure
  - Consists of connected components
  - Components run network services
  - Defender defends the infrastructure by monitoring and active defense
  - Has partial observability
- An **Attacker** seeks to intrude on the infrastructure
  - Has a partial view of the infrastructure
  - Wants to compromise specific components
  - Attacks by reconnaissance, exploitation and pivoting



The Intrusion Prevention Problem



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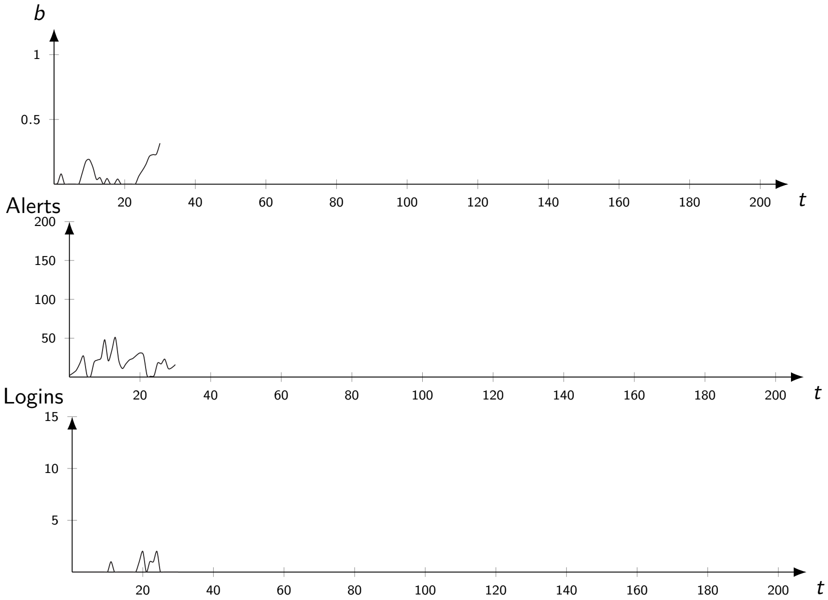

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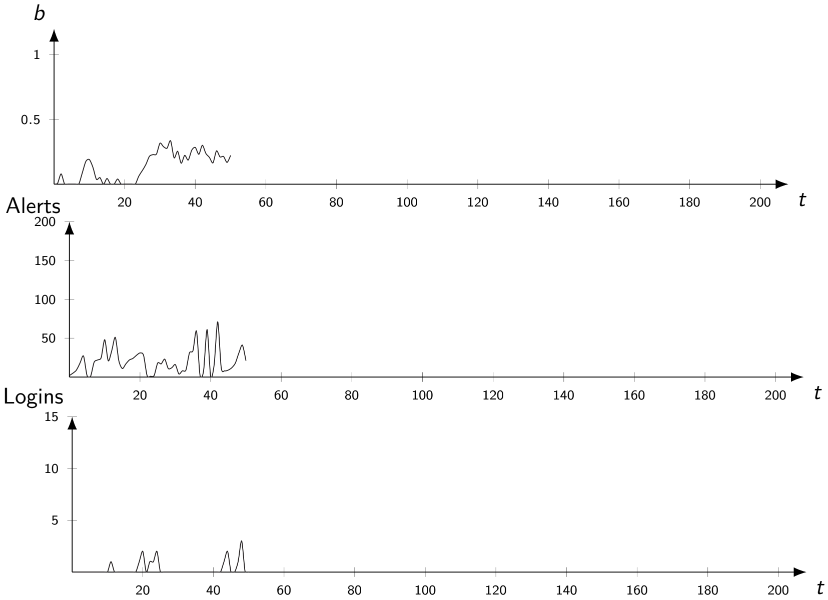

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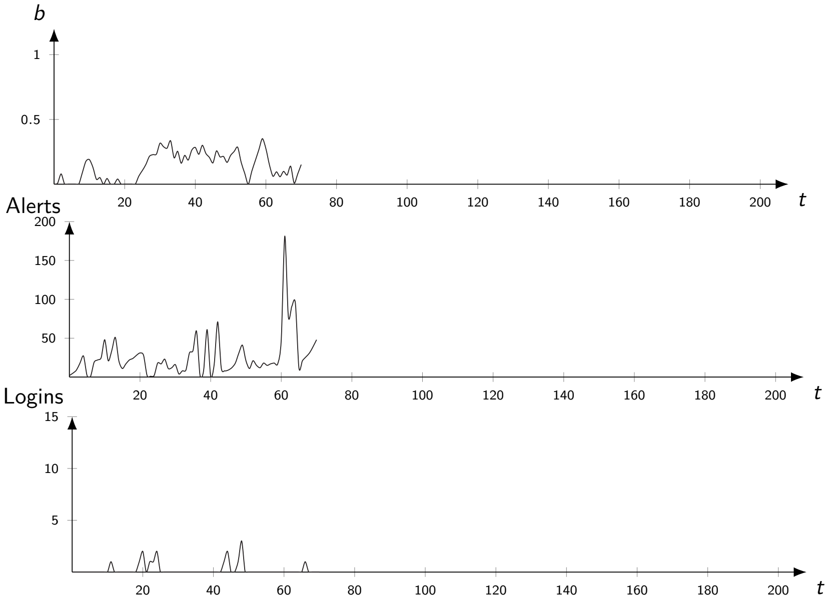

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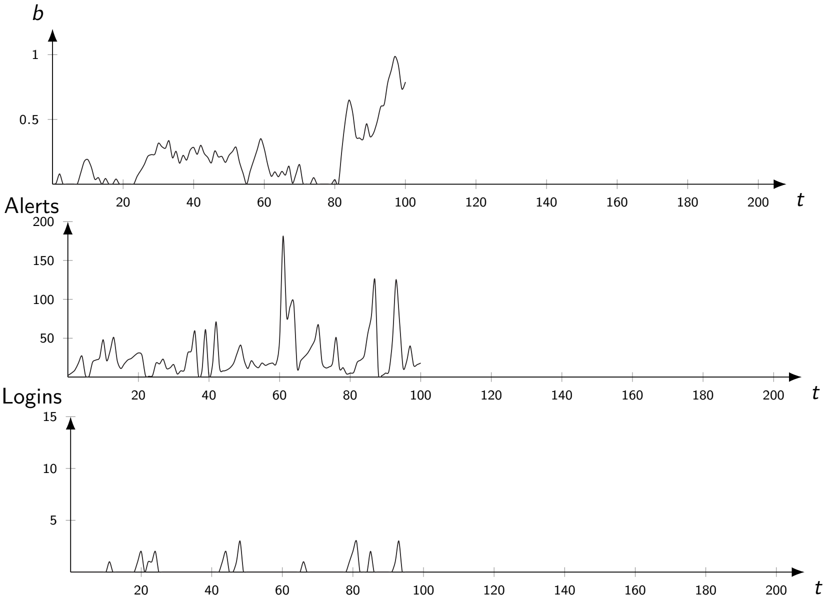

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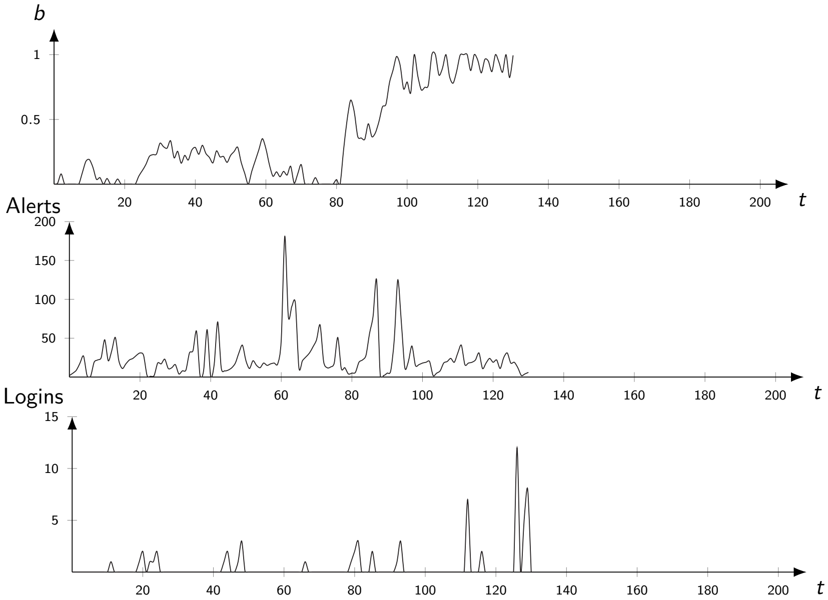

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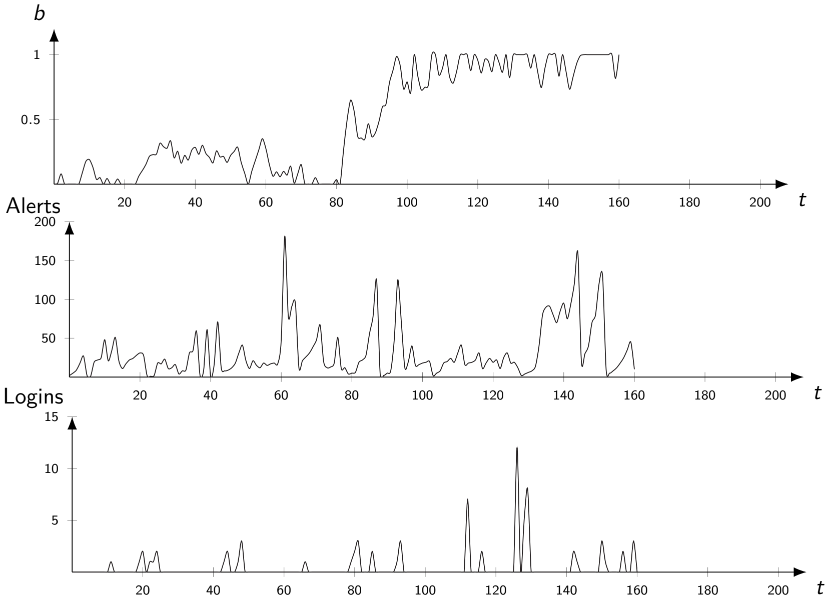

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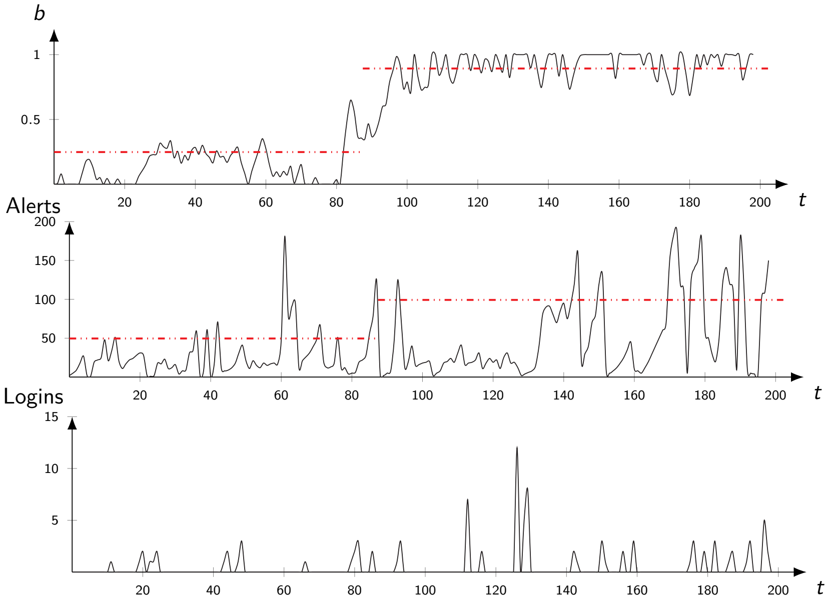
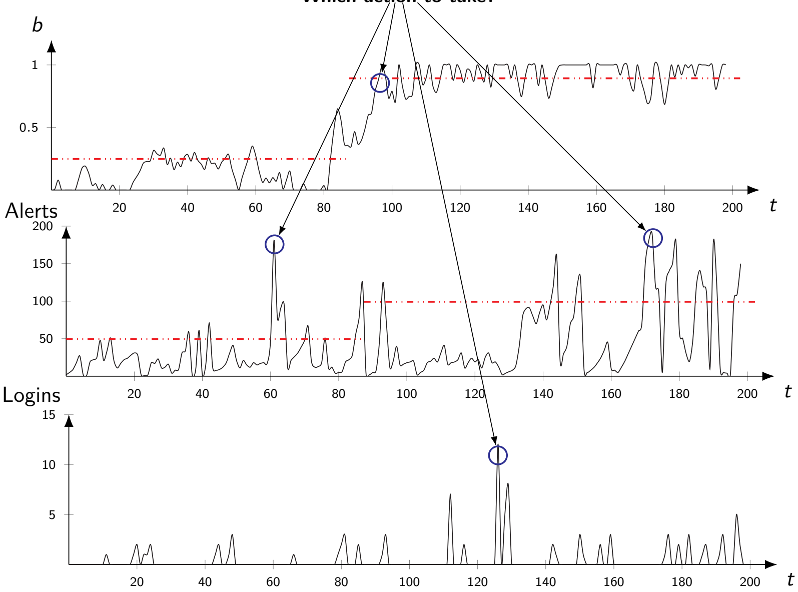


The Intrusion Prevention Problem

When to take a defensive action?
Which action to take?



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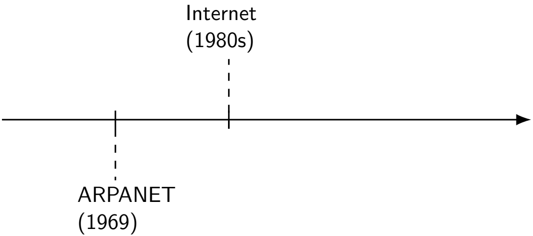

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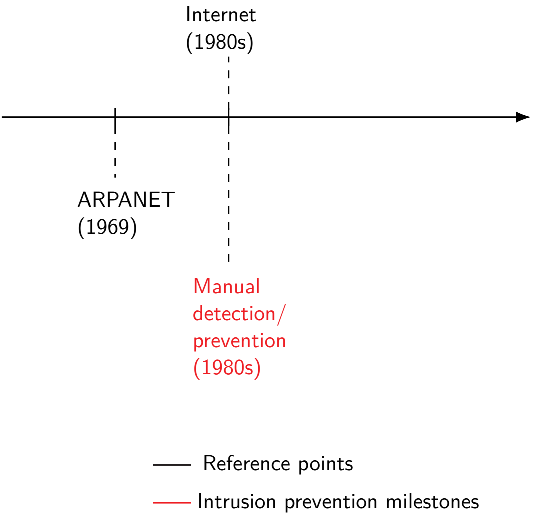

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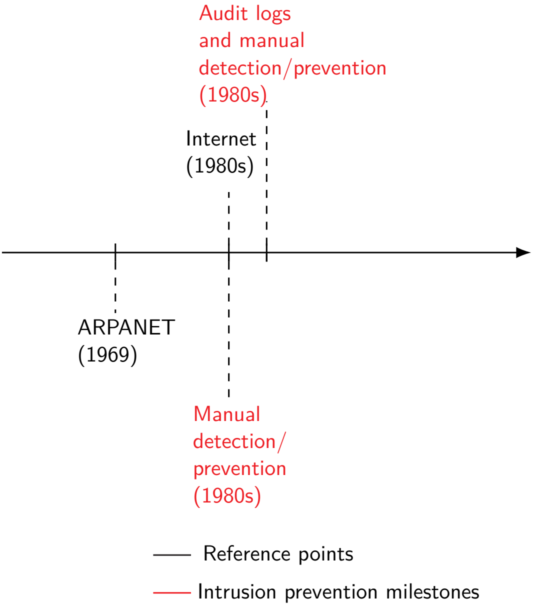

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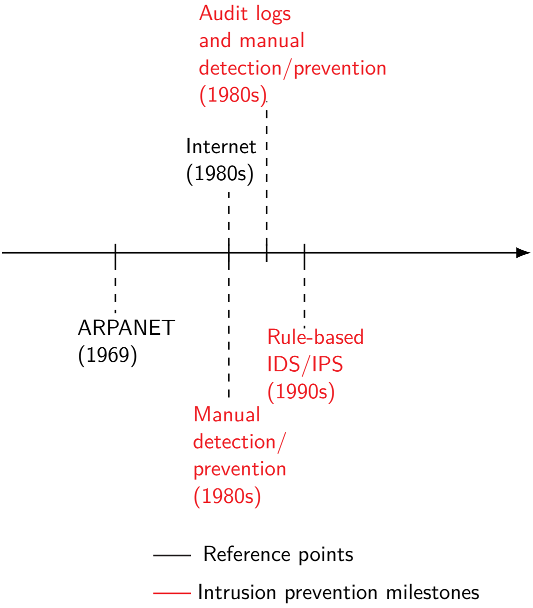

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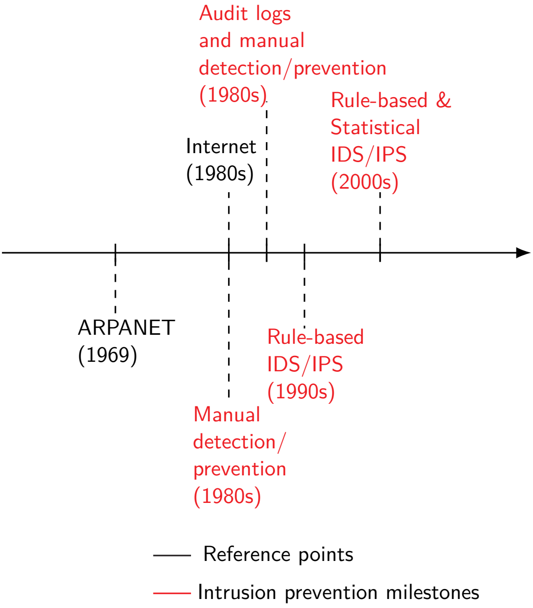

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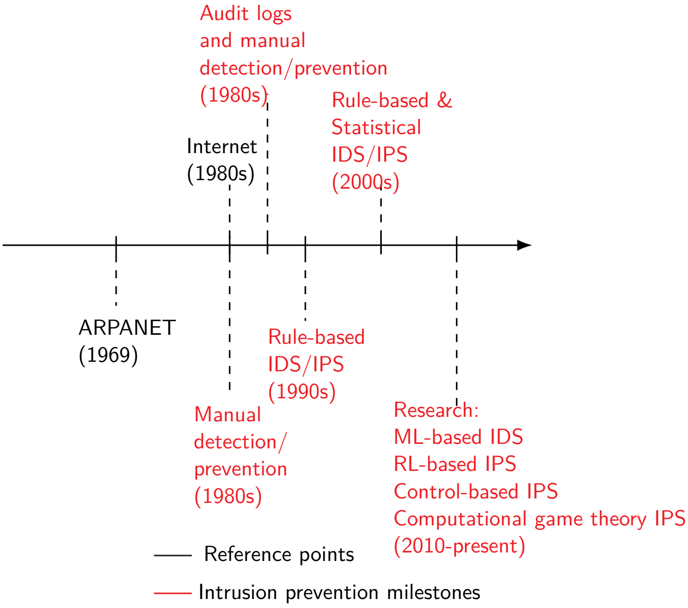
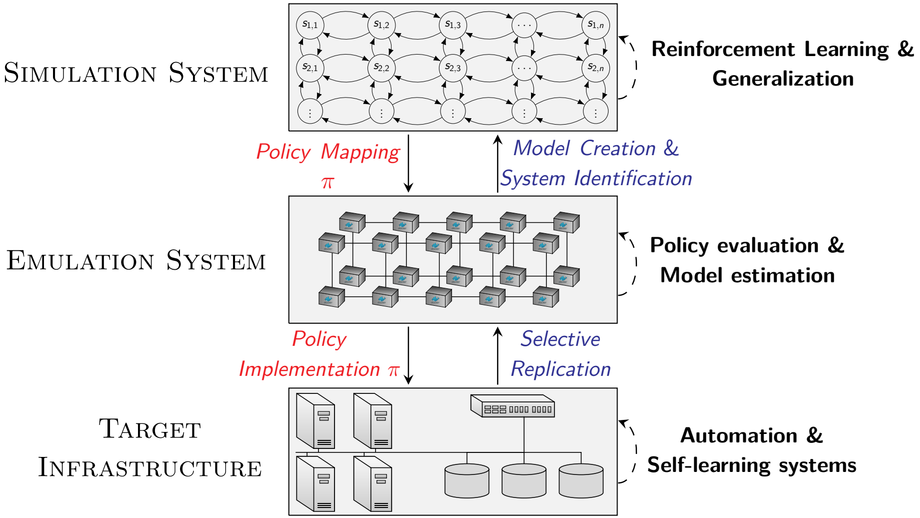


Our Method for Learning Effective Security Strategies



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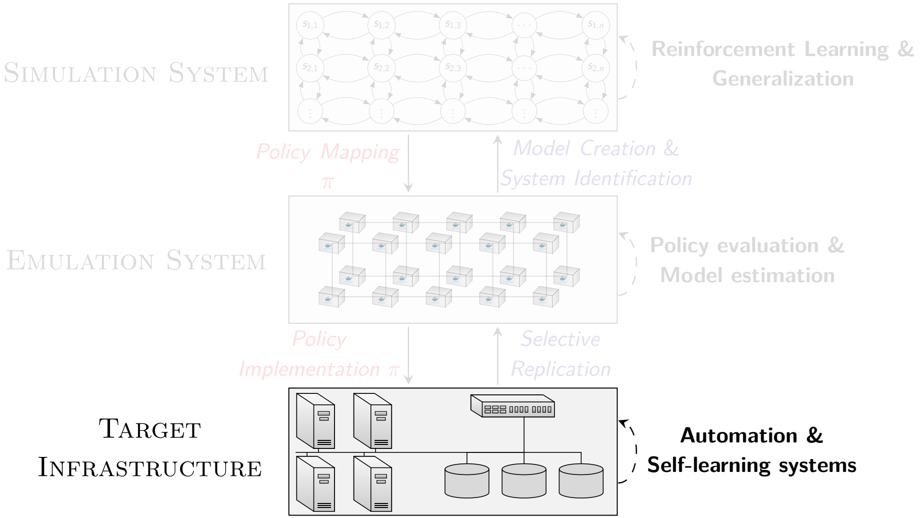

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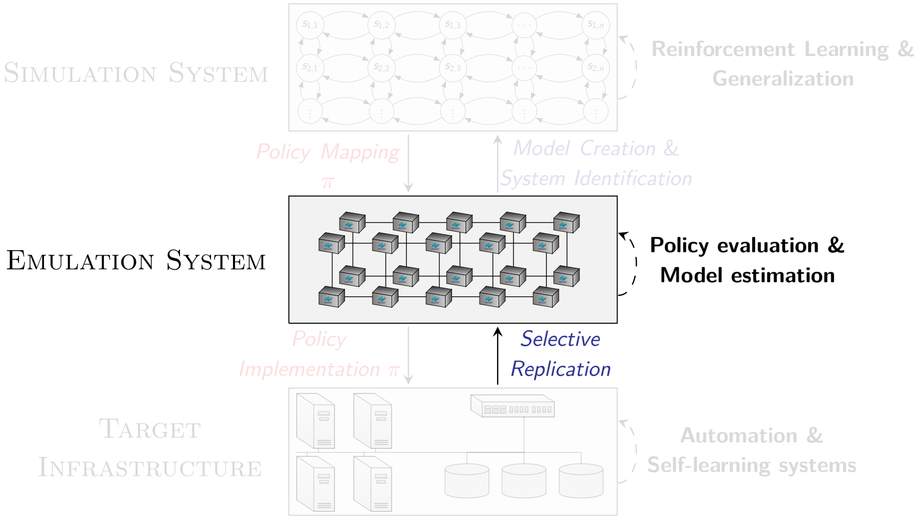

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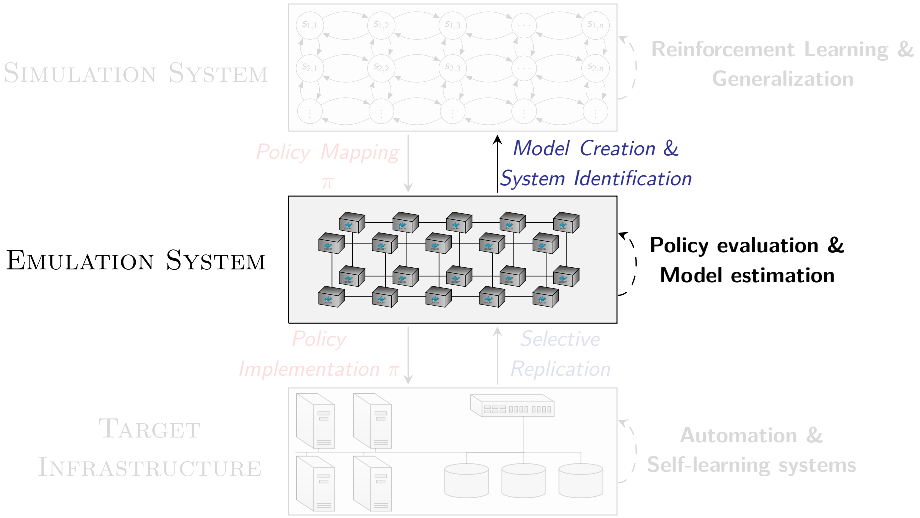

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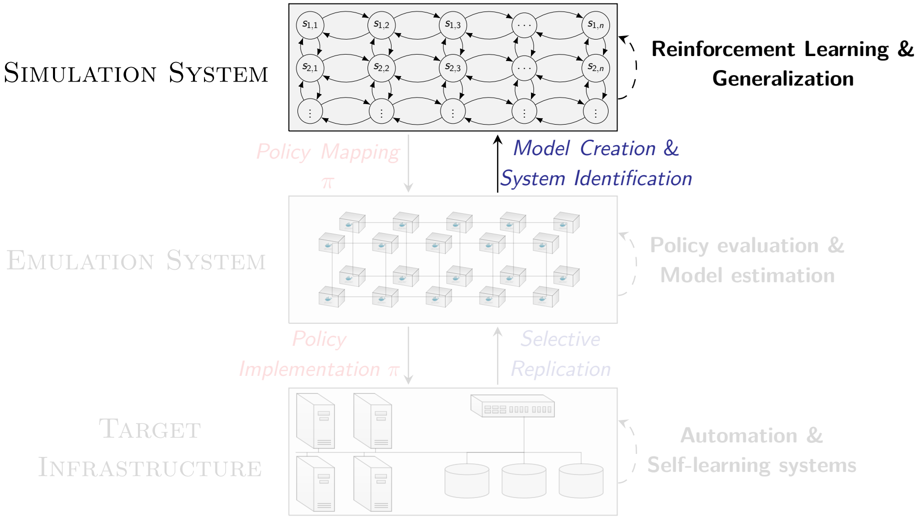

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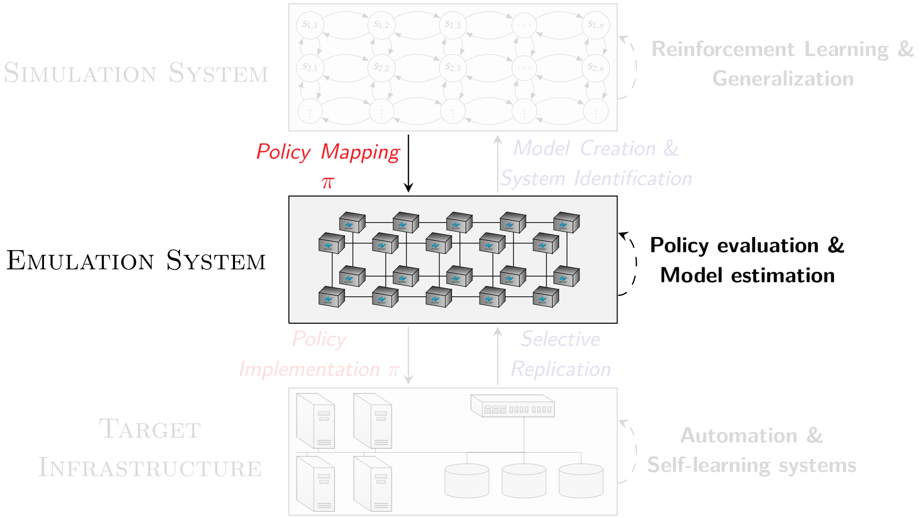

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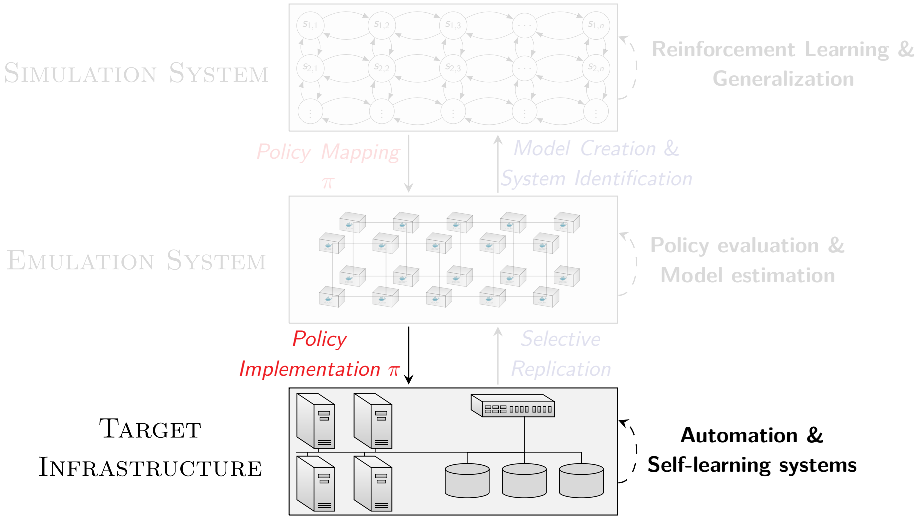

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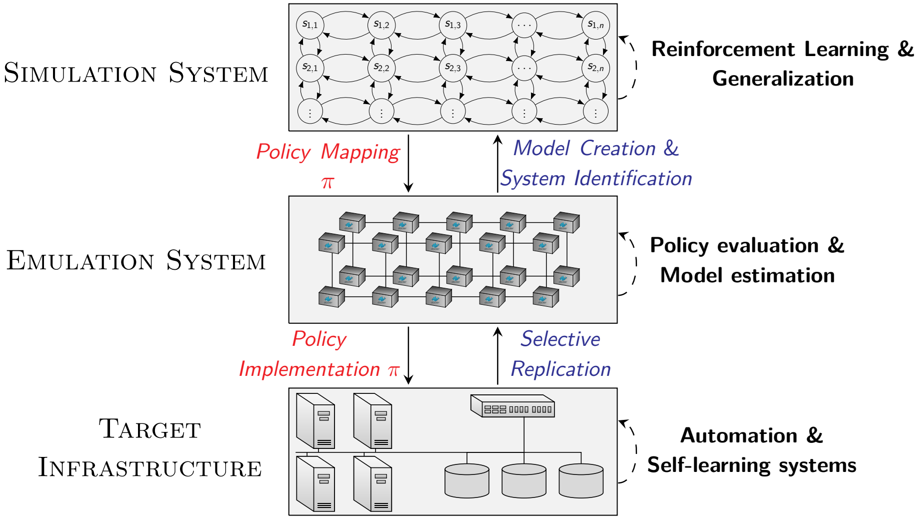

Outline

- **Use Case & Approach:**
  - Intrusion prevention
  - System identification
  - Reinforcement learning and optimal stopping
- **Formal Model of The Use Case**
  - Intrusion prevention as an optimal stopping problem
  - Partially observed Markov decision process
- **Structure of π^***
  - Existence of optimal multi-threshold policy π_l^*
  - Stopping sets $\mathscr{S}_l$ are connected and nested
- **Reinforcement learning method**
  - Learning threshold policies & the policy gradient
  - Emulated infrastructure
- **Results & Conclusion**
  - Numerical evaluation results & Demo
  - Conclusion & future work

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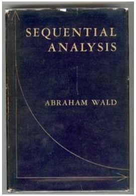
Background: Optimal Stopping

- **History**:
  - Studied in the 18th century to analyze a gambler's fortune
  - Formalized by Abraham Wald in 1947[1]
  - Since then it has been generalized and developed by (Chow[2], Shiryaev & Kolmogorov[3], Bather[4], Bertsekas[5], etc.)



[1] Abraham Wald. *Sequential Analysis*. Wiley and Sons, New York, 1947.

[2] Y. Chow, H. Robbins, and D. Siegmund. "Great expectations: The theory of optimal stopping". In: 1971.

[3] Albert N. Shirayev. *Optimal Stopping Rules*. Reprint of russian edition from 1969. Springer-Verlag Berlin, 2007.

[4] John Bather. *Decision Theory: An Introduction to Dynamic Programming and Sequential Decisions*. USA: John Wiley and Sons, Inc., 2000. ISBN: 0471976490.

[5] Dimitri P. Bertsekas. *Dynamic Programming and Optimal Control*. 3rd. Vol. I. Belmont, MA, USA: Athena Scientific, 2005.

Background: Optimal Stopping

- **The General Problem**:
  - A stochastic process $(s_t)_{t=1}^{T}$ is observed sequentially
  - Two options per t: (i) continue to observe; or (ii) stop
  - Find the *optimal stopping time* τ^*:

$$\tau^* = \arg\max_{\tau} \mathbb{E}_{\tau}\left[\sum_{t=1}^{\tau-1} \gamma^{t-1}\mathcal{R}^{C}_{s_t s_{t+1}} + \gamma^{\tau-1}\mathcal{R}^{S}_{s_\tau s_\tau}\right] \qquad (1)$$

where $\mathcal{R}^{S}_{ss'}$ & $\mathcal{R}^{C}_{ss'}$ are the stop/continue rewards

Background: Optimal Stopping

- **Applications & Use Cases**:
  - Hypothesis testing[6]
  - Change detection[7],
  - Selling decisions[8],
  - Queue management[9],
  - Industrial control[10],
  - Advertisement scheduling[11], etc.

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[6] Abraham Wald. *Sequential Analysis*. Wiley and Sons, New York, 1947.

[7] Alexander G. Tartakovsky et al. "Detection of intrusions in information systems by sequential change-point methods". In: *Statistical Methodology* (2006). ISSN: 1572-3127. DOI: https://doi.org/10.1016/j.stamet.2005.05.003. URL: https://www.sciencedirect.com/science/article/pii/S1572312705000493.

[8] Jacques du Toit and Goran Peskir. "Selling a stock at the ultimate maximum". In: *The Annals of Applied Probability* 19.3 (2009). ISSN: 1050-5164. DOI: 10.1214/08-aap566. URL: http://dx.doi.org/10.1214/08-AAP566.

[9] Arghyadip Roy et al. "Online Reinforcement Learning of Optimal Threshold Policies for Markov Decision Processes". In: *CoRR* (2019). http://arxiv.org/abs/1912.10325. eprint: 1912.10325.

[10] Maben Rabi and Karl H. Johansson. "Event-Triggered Strategies for Industrial Control over Wireless Networks". In: *Proceedings of the 4th Annual International Conference on Wireless Internet*. WICON '08. Maui, Hawaii, USA, 2008. ISBN: 9789639799363.

[11] Vikram Krishnamurthy, Anup Aprem, and Sujay Bhatt. "Multiple stopping time POMDPs: Structural results & application in interactive advertising on social media". In: *Automatica* 95 (2018), pp. 385–398. ISSN: 0005-1098. DOI: https://doi.org/10.1016/j.automatica.2018.06.013. URL: https://www.sciencedirect.com/science/article/pii/S0005109818303054.

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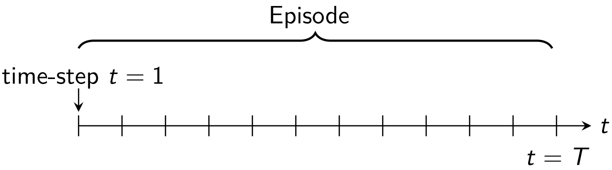
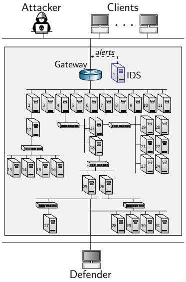
[17] Kim Hammar and Rolf Stadler. "Intrusion Prevention through Optimal Stopping". In: (). 2021, https://arxiv.org/abs/2111.00289. arXiv: 2111.00289.

Formulating Intrusion Prevention as a Stopping Problem





- The system evolves in discrete time-steps.
- Defender observes the infrastructure (IDS, log files, etc.).
- An intrusion occurs at an unknown time.
- The defender can make L stops.
- Each stop is associated with a defensive action
- The final stop shuts down the infrastructure.
- **Based on the observations, when is it optimal to stop?**

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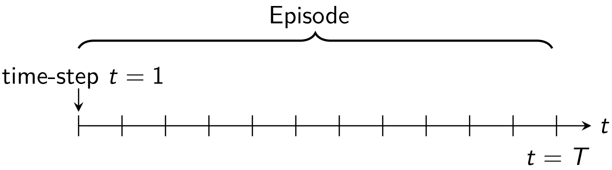

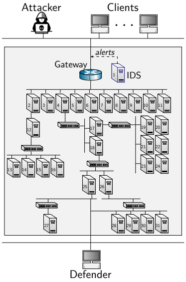

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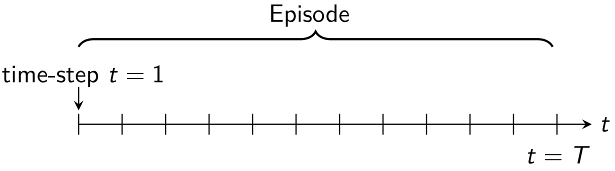

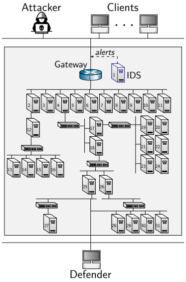

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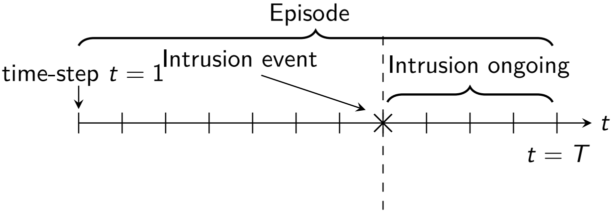

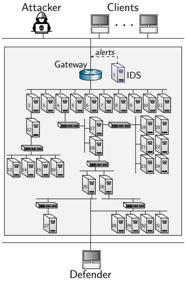

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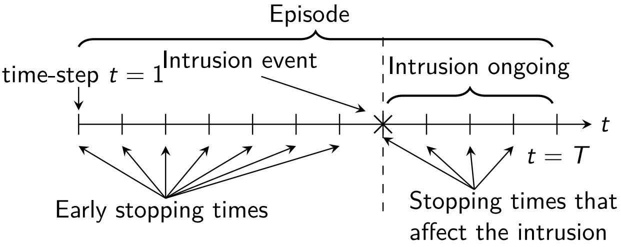

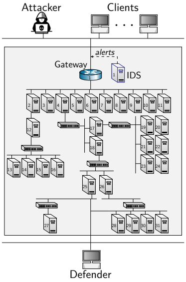

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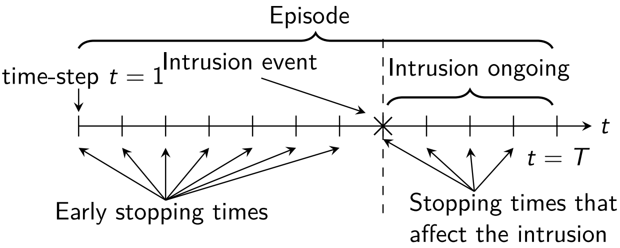

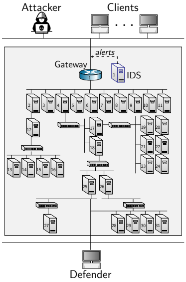

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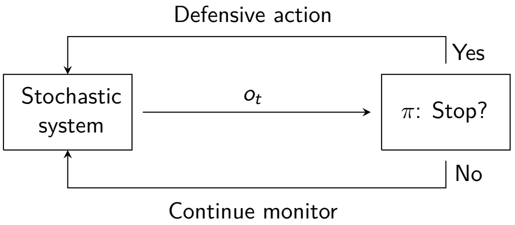


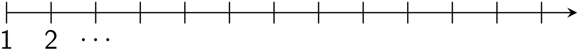
- The $L-l$th **stopping time** τ_l is:

$$\tau_l = \inf\{t : t > \tau_{l-1}, a_t = S\}, \qquad l \in 1, .., L, \ \tau_{L+1} = 0$$

- τ_l is a random variable from sample space Ω to $\mathbb{N}$, which is dependent on $h_{\tau_l} = \rho_1, a_1, o_1, \ldots, a_{\tau_l - 1}, o_{\tau_l}$ and independent of $a_{\tau_l}, o_{\tau_l + 1}, \ldots$

Intrusion Prevention through Optimal Stopping

We consider the class of stopping times
$\mathcal{T}_t = \{\tau_l \leq t | \tau_l > \tau_{l-1}\} \in \mathcal{F}_k$ ($\mathcal{F}_k$ =natural filtration on h_t).



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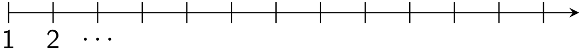

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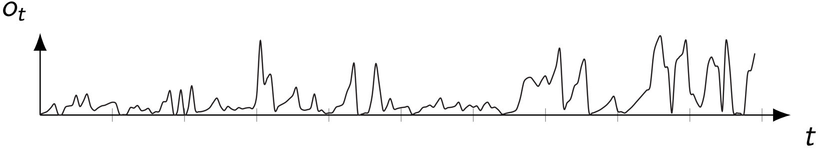

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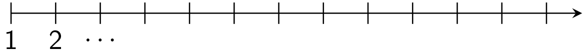

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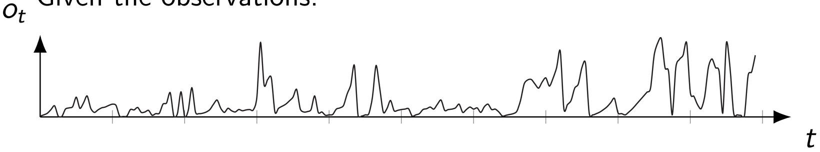
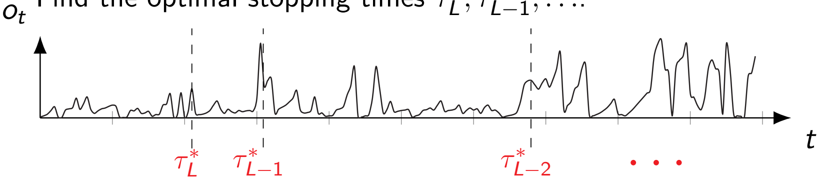
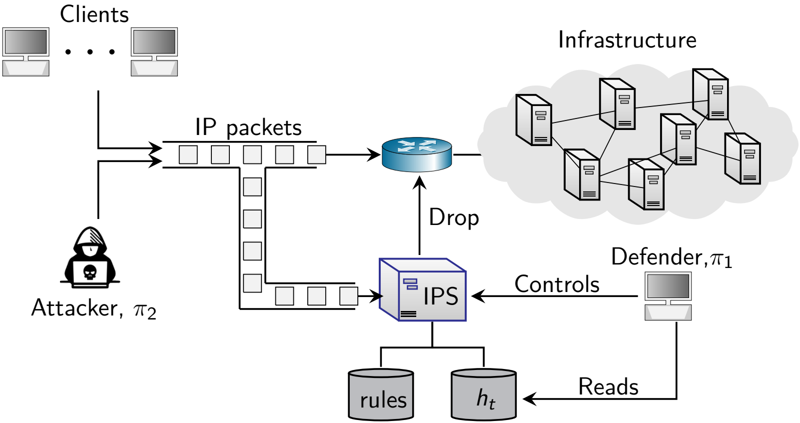


Find the optimal stopping times $\tau^*_L, \tau^*_{L-1}, \ldots$:



The Defender's Stop Actions

- Ingress traffic goes through deep packet inspection at gateway
- Gateway runs the Snort IDS/IPS and may drop packets
- At each stopping time, we update the IPS configuration



The Defender's Stop Actions

- Ingress traffic goes through deep packet inspection at gateway
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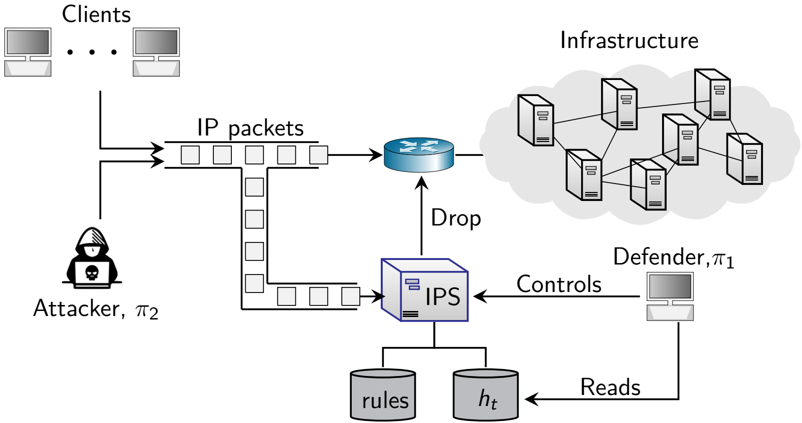
- **Objective:** find optimal π^* or Nash equilibrium



Approaches to Solving Optimal Stopping Problems

Two main approaches:

- The *Markovian approach*
  - Assume process is Markov
  - Utilize Markov deision theory
- The *martingale approach*
  - More general
  - No Markov assumption
  - Utilize martingale convergence theorems

The Markovian Approach to Optimal Stopping

- Model the problem as a MDP or POMDP
- A policy π^* that satisfies the Bellman-Wald equation is optimal:

$$\pi^*(s) = \arg\max_{\{S,C\}} \left[\underbrace{\mathbb{E}\left[\mathcal{R}_s^S\right]}_{\text{stop}}, \underbrace{\mathbb{E}\left[\mathcal{R}_s^C + \gamma V^*(s')\right]}_{\text{continue}} \right] \quad \forall s \in \mathcal{S}$$

- Solve by backward induction, dynamic programming, or reinforcement learning

- **Alternative optimality condition**:
  - Theorem: $V^*(s)$ is the minimal excessive function which majorizes $R_s^{\emptyset}$.
  - Assume all rewards are received upon stopping: $R_s^{\emptyset}$
  - $V^*(s)$ **majorizes** $R_s^{\emptyset}$ if $V^*(s) \geq R_s^{\emptyset}$ $\forall s \in \mathcal{S}$
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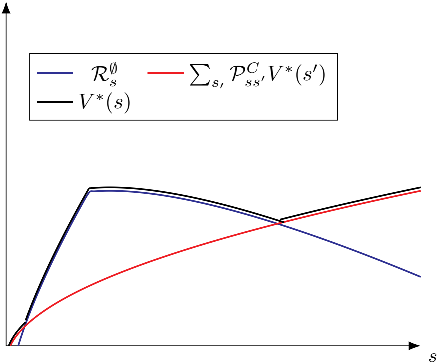

The Martingale Approach to Optimal Stopping

- Model the state process as an arbitrary stochastic process
- The reward of the optimal stopping time is given by the Snell envelope[18].
- Snell envelope: **smallest supermartingale that stochastically dominates the process**

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We follow the Markovian approach and model the problem as a POMDP

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A Partially Observed MDP Model for the Defender

- **States:**
  - Intrusion state $s_t \in \{0,1\}$, terminal $\emptyset$.
- **Observations:**
  - Severe/Warning IDS Alerts $(\Delta x, \Delta y)$, Login attempts Δz, stops remaining $l_t \in \{1,..,L\}$, $f_{XYZ}(\Delta x, \Delta y, \Delta z | s_t)$
- **Actions:**
  - "Stop" (S) and "Continue" (C)
- **Rewards:**
  - Reward: security and service. Penalty: false alarms and intrusions
- **Transition probabilities:**
  - Bernoulli process $(Q_t)_{t=1}^{T} \sim Ber(p)$ defines intrusion start $I_t \sim Ge(p)$
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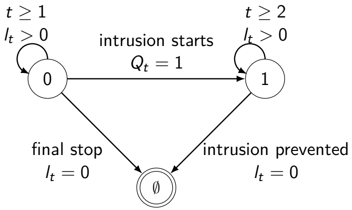

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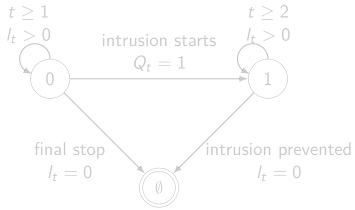

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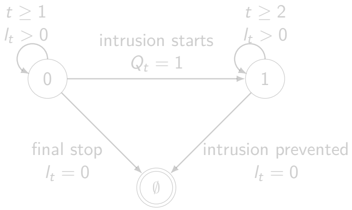

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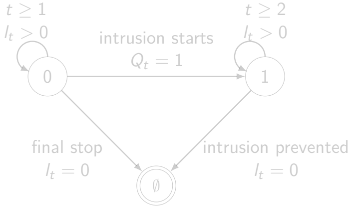

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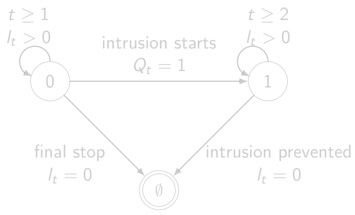

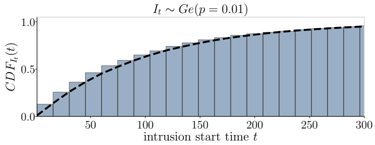

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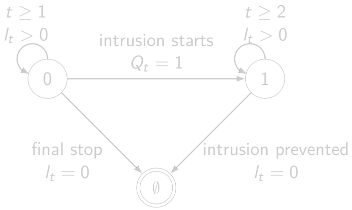

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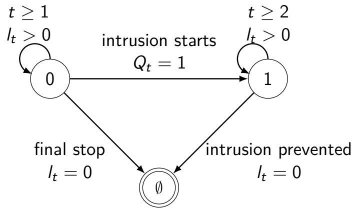

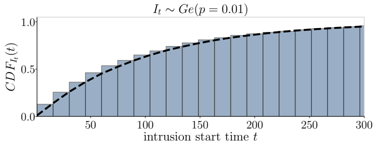

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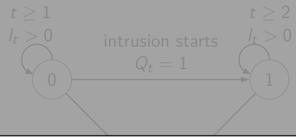

We analyze the structure of π^* using POMDP & stopping theory

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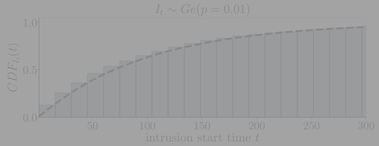

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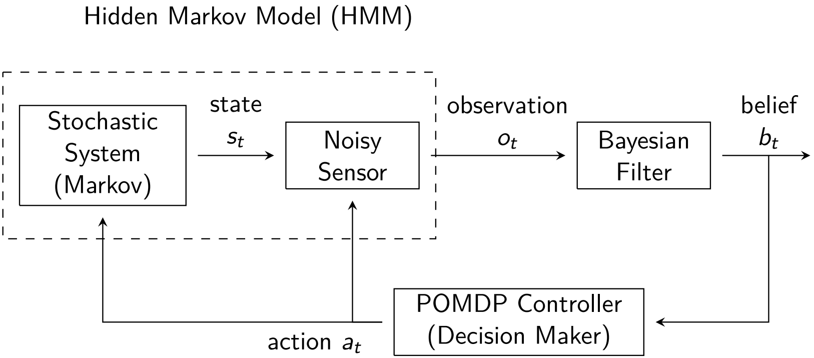
Outline

- **Use Case & Approach:**
  - Intrusion prevention
  - System identification
  - Reinforcement learning and optimal stopping
- **Formal Model of The Use Case**
  - Intrusion prevention as an optimal stopping problem
  - Partially observed Markov decision process
- **Structure of π^***
  - Existence of optimal multi-threshold policy π_l^*
  - Stopping sets $\mathscr{S}_l$ are connected and nested
- **Reinforcement learning method**
  - Learning threshold policies & the policy gradient
  - Emulated infrastructure
- **Results & Conclusion**
  - Numerical evaluation results & Demo
  - Conclusion & future work

Background: POMDPs



- **POMDP:** $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}^{a_t}_{s_t, s_{t+1}}, \mathcal{R}^{a_t}_{s_t, s_{t+1}}, \gamma, \rho_1, T, \mathcal{O}, \mathcal{Z} \rangle$
- Controlled hidden Markov model, states $s_t \in \mathcal{S}$ are hidden
- Agent observes history $h_t = (\rho_1, a_1, o_1, \ldots, a_{t-1}, o_t) \in \mathcal{H}$

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- **Optimality (Bellman) Eq:**

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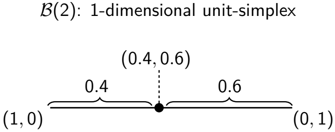

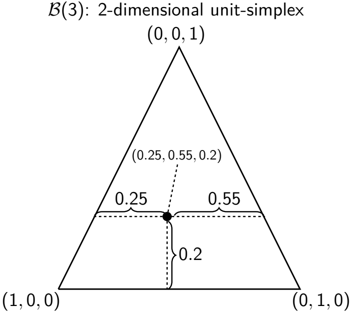

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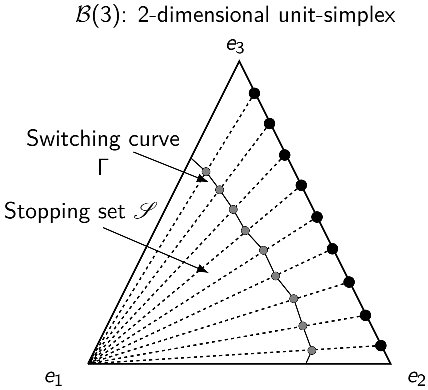

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Structural Result: Optimal Multi-Threshold Policy

Theorem

Given the intrusion prevention POMDP, the following holds:

1. $\mathscr{S}_{l-1} \subseteq \mathscr{S}_l$ *for* $l = 2, \dots L$.
2. *If* $L = 1$, *there exists an optimal threshold* $\alpha^* \in [0, 1]$ *and an optimal policy of the form:*

$$\pi_L^*(b(1)) = S \iff b(1) \geq \alpha^* \tag{2}$$

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where α_l^* *is decreasing in* l.

Structural Result: Optimal Multi-Threshold Policy

Theorem

Given the intrusion prevention POMDP, the following holds:

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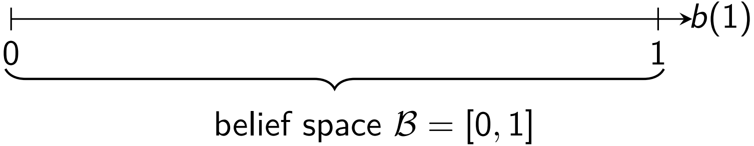

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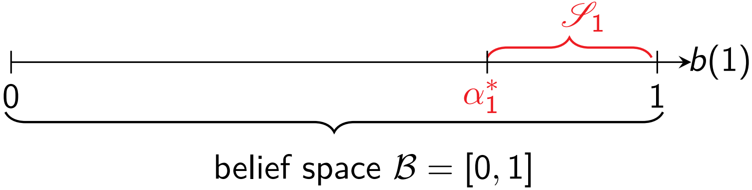

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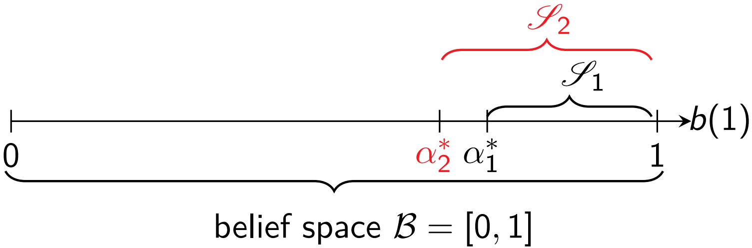

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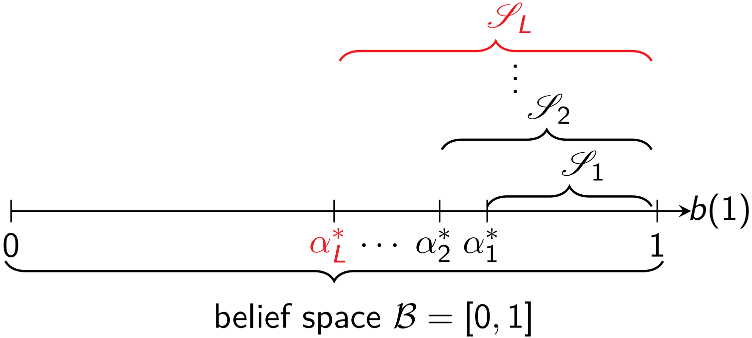

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Our Reinforcement Learning Algorithm for Learning Threshold Policies

- We **use the structural result** that an optimal threshold policy exist (Theorem 1) **to design an efficient reinforcement learning algorithm**.
- We seek to learn L thresholds: $\alpha_1^*, \alpha_2^*, \ldots, \alpha_L^*$
- We learn these thresholds iteratively through Robbins and Monro's stochastic approximation algorithm.[20]

[20] Herbert Robbins and Sutton Monro. "A Stochastic Approximation Method". In: *The Annals of Mathematical Statistics* 22.3 (1951), pp. 400 –407. DOI: 10.1214/aoms/1177729586. URL: https://doi.org/10.1214/aoms/1177729586.

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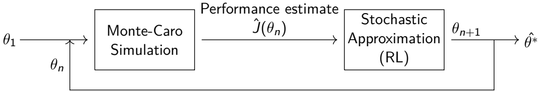

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Our Reinforcement Learning Algorithm for Learning Threshold Policies

1. Parameterize the policy $\pi_{I,\theta^{(1)}}$ by $\theta \in \mathbb{R}^L$
2. The *policy gradient*

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_{I,\theta}}\left[\sum_{t=1}^{\infty} \nabla_\theta \log \pi_{I,\theta}(a_t|s_t) \sum_{\tau=t}^{\infty} r_t\right]$$

   exists as long as $\pi_{I,\theta}$ is differentiable.
3. A pure threshold policy is not differentiable.
4. To ensure differentiability and to constrain the thresholds to be in $[0, 1]$, we define $\pi_{\theta,I}$ to be a smooth stochastic policy that approximates a threshold policy:

$$\pi_{i,\theta}(S|b(1)) = \left(1 + \left(\frac{b(1)(1-\sigma(\theta_l))}{\sigma(\theta_l)(1-b(1))}\right)^{-20}\right)^{-1}$$

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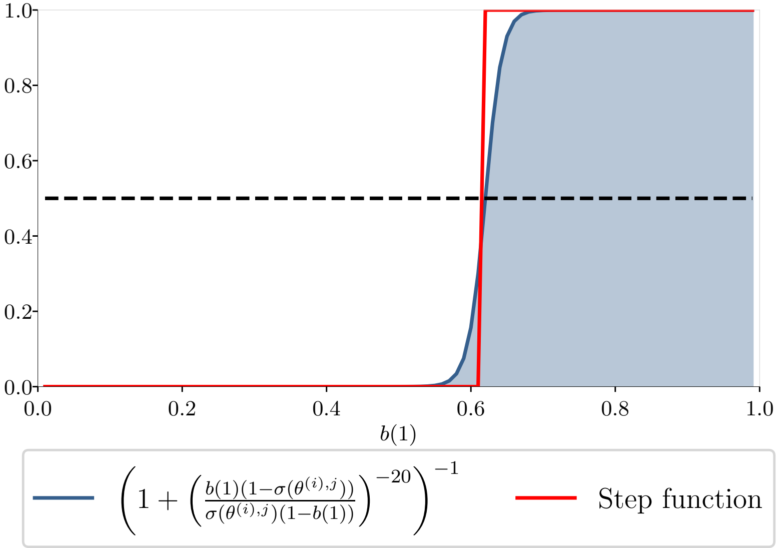
Smooth Threshold Policy



Our Reinforcement Learning Algorithm for Learning Threshold Policies

1. We learn the thresholds through simulation.
2. For each iteration $n \in \{1, 2, \ldots\}$, we perturb θ_n to obtain $\theta_n + c_n\Delta_n$ and $\theta_n - c_n\Delta_n$.
3. Then, we simulate two POMDP episodes
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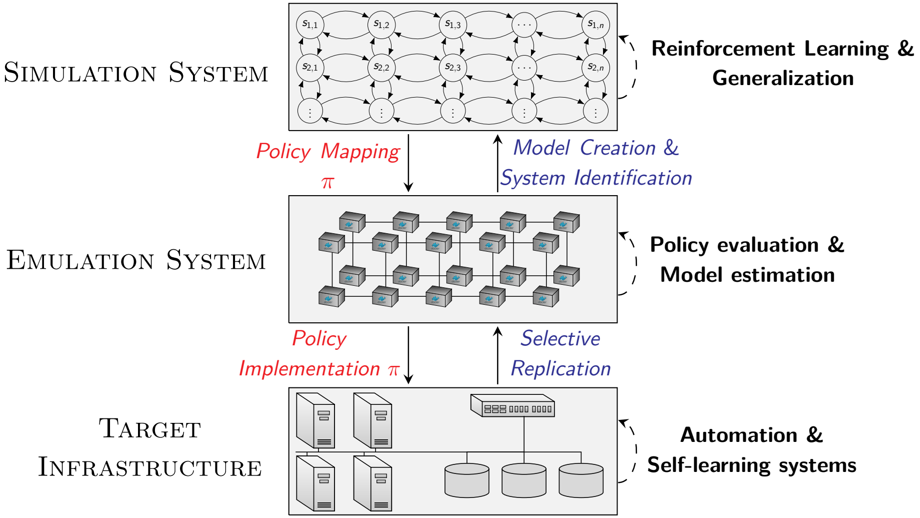
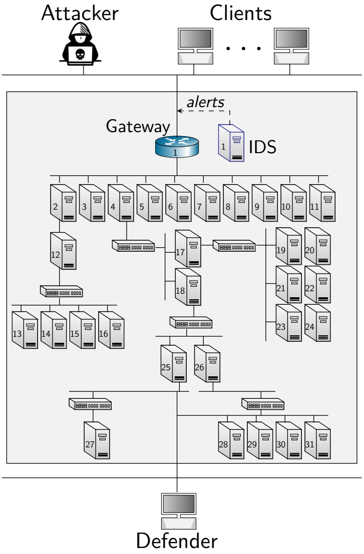
To evaluate Policies Learned in Simulation we Run them in the Emulation



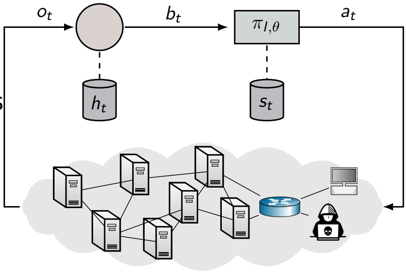
Emulating the Target Infrastructure

- Emulate **hosts** with docker containers
- Emulate **IDS and vulnerabilities** with software
- Network isolation and **traffic shaping** through NetEm in the Linux kernel
- Enforce **resource constraints** using cgroups.
- Emulate **client arrivals** with Poisson process
- **Internal connections** are full-duplex & loss-less with bit capacities of 1000 Mbit/s
- **External connections** are full-duplex with bit capacities of 100 Mbit/s & 0.1% packet loss in normal operation and random bursts of 1% packet loss



Running a POMDP Episode in the Emulation

- A distributed system with synchronized clocks
- We run software sensors on all emulated hosts
- Sensors produce messages to a distributed queue (Kafka)
- A stream processor (Spark) consumes messages from the queue and computes statistics
- Actions are selected based on the computed statistics and the policies
- Actions are sent to the emulation using gRPC
- Actions are executed by running commands on the hosts



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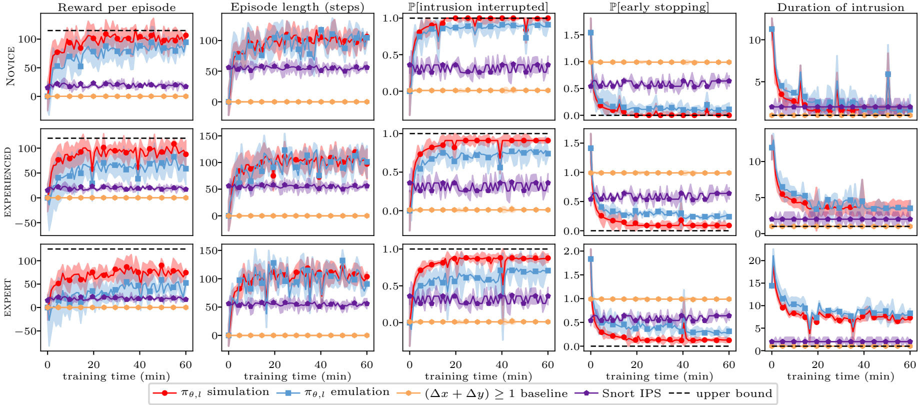
Evaluation Results



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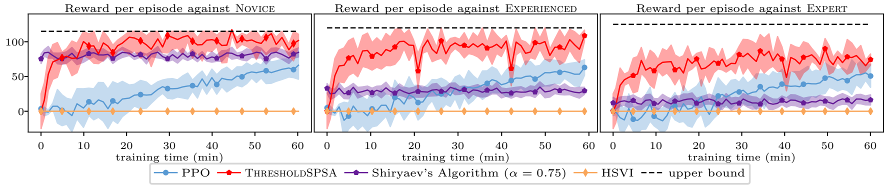
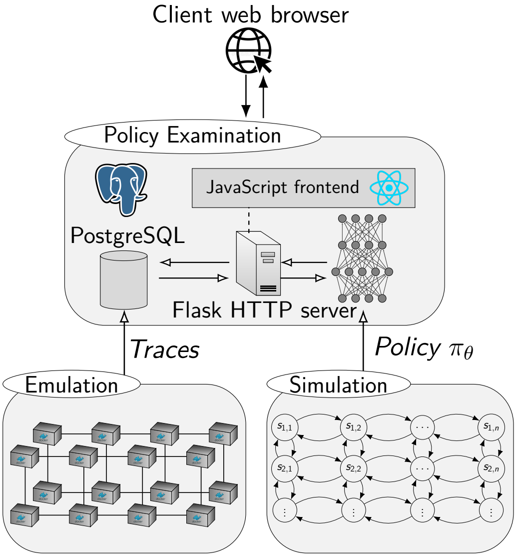


Demo - A System for Interactive Examination of Learned Security Policies



Architecture of the system for examining learned security policies.

Conclusions & Future Work

- **Conclusions:**
  - We develop a *method* to automatically learn security policies
    - (1) emulation system; (2) system identification; (3) simulation system; (4) reinforcement learning and (5) domain randomization and generalization.
  - We apply the method to an **intrusion prevention use case**
  - We formulate intrusion prevention as a multiple stopping problem
    - We present a POMDP model of the use case
    - We apply the stopping theory to establish structural results of the optimal policy
    - We design a reinforcement learning algorithm that outperforms state-of-the-art on our use case
    - We show numerical results in realistic emulation environment
- **Our research plans:**
  - Extending the model
    - Active attacker: Partially Observed Stochastic Game, Equilibrium analysis
    - Less restrictions on defender